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Reflexives and non-Fregean quantifiers

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It is shown that depending on the subject noun phrase sequences of noun phrases and reflexive expressions give rise to three formally different type $\langle 2 \rangle$ quantifiers. If the noun phrase is a proper name, the quantifier is reducible, if the noun phrase denotes a filter, the quantifier is weakly reducible and if the noun phrase denotes an atom, the corresponding quantifier is strongly irreducible

Introduction

Natural languages display a great variety of constructions which denote non-Freagean quantifiers that is complex quantifiers which are not iterations of simpler quantifiers. Such quantifiers have been extensively studied by Ed (Clark and Keenan (1986), Keenan (1987b), Keenan (1992)). Usually expressions denoting non-Fregean quantifiers are not lexically simple and often they are not syntactic constituents. They are sequences composed of an NP (DP) and of an expression which can be called a generalised NP (GNP). GNPs are linguistic objects that can play the role of syntactic arguments of transitive VPs. So "ordinary" NPs or DPs are GNPs. However there are *genuine* GNPs which differ from "ordinary" NPs in that they cannot play the role of all verbal arguments; in particular they cannot occur in subject position. For instance the reflexive pronoun *himself/herself* or the reciprocal *each other* are such genuine GNPs.

A sequence NP...GNP can be considered as applying to a transitive VP to give a sentence S. In this case such sequence denotes a type $\langle 2 \rangle$ quantifier. If the GNP is a genuine GNP then often the sequence NP...GNP denotes a non-Fregean type $\langle 2 \rangle$ quantifier. In this note I present some general results concerning non-Fregean quantifiers denoted by the sequence NP...GNP in case when GNP is the reflexive pronoun or a Boolean compound of the reflexive pronoun and another expression. Thus, roughly speaking, I will show that the type $\langle 2 \rangle$ quantifiers involved in the interpretation of the following examples have different properties:

(1) Leo washed himself/himself and Lea.

(2) Leo and Lea/every student washed themselves

(3) Only Leo washed himself.

The GNPs that I will consider are those which denote specific type $\langle 2 : 1 \rangle$ functions (functions from binary relations to sets). An ordinary NP occurring in (direct) object position in a sentence can also be considered as denoting a type $\langle 2 : 1 \rangle$ function. When occurring

in subject position in a simple '"intransitive" sentence, an NP denotes a type $\langle 1 \rangle$ quantifier, that is a function from sets to truth values. When occurring in direct object position this NP denotes the *accusative extension* of the quantifier denoted by this NP on subject position. The accusative extension Q_{acc} (which is a function from binary relations to sets) of the quantifier Q is defined as follows (Keenan (1987a):

Definition 1. $Q_{acc}(R) = \{x : (QxR) = 1\}$, where $xR = \{y : \langle x, y \rangle \in R\}$

Type $\langle 2:1 \rangle$ functions which are accusative extensions of some type $\langle 1 \rangle$ quantifier satisfy so-called *case extension condition* EC:

Definition 2. A type $\langle 2:1 \rangle$ function F satisfies EC iff for an y $a, b \in E$ and any binary relation R and S , if $aR = bS$ then $a \in F(R)$ iff $b \in F(S)$.

Here basically functions which satisfy a weaker condition, so-called *predicate invariance* (Keenan and Westerståhl (1997)) are considered:

Definition 3. A type $\langle 2:1 \rangle$ function F is *predicate invariant* iff for any $a \in E$ and any binary relations R, S, if $aR = aS$ then $a \in F(R)$ iff $a \in F(S)$, where R, S are binary relations, E is the universe and $aR = \{x : \langle a, x \rangle \in R\}$.

Obviously functions satisfying EC are predicate invariant. It is important to observe, however (cf. Keenan (2007)), that functions denoted by genuine GNPs (like reflexive pronouns or by many expressions which are Boolean compounds of them), do not satisfy EC, even if they are predicate invariant. Similarly, functions denoted by GNPs formed from anaphoric determiners such as *Every...except himself, Most...,including herself* or specific possessive anaphoric determiners found in Slavic languages for instance, do not satisfy EC (Zuber 2010) . Thus *SELF* does not satisfy EC and, moreover, for any type $\langle 1 \rangle$ quantifier Q the type $\langle 2:1 \rangle$ function $F = SELF \oplus Q_{acc}$, where $\oplus$ is a binary Boolean operator, is a predicate invariant function which does not satisfy EC. Similarly the function $NO(A)$-BUT-$SELF$, as specified in (4), does not satisfy the EC for any $A \neq \emptyset$ (Zuber 2010):

(4) $NO(A)\text{-}BUT\text{-}SELF(R) = \{x : A \cap xR = \{x\}\}$

Thus the difference between genuine GNPs (considered here) and NPs is that the former denote predicate invariant functions which do not satisfy EC.

1 Reflexives and Fregean quantifiers

A set of binary relations is a type $\langle 2 \rangle$ quantifier; among them one can distinguish the following sub-class (cf. Keenan (1992)):

Definition 4. A type $\langle 2 \rangle$ quantifier F is Fregean, or Frege reducible iff there exist two type $\langle 1 \rangle$ quantifiers Q and Q_1 such that $F(R) = Q_1(Q_{acc}(R))$.

A type $\langle 2 \rangle$ quantifier is non-Fregean iff it is not Frege reducible.

Various tests showing that a type $\langle 2 \rangle$ quantifier is Fregean have been established and various type $\langle 2 \rangle$ quantifiers have been shown to be non-Fregean (Keenan (1992), Ben Shalom (1994), (van Eijck 2005)) with their help. In these tests essential role play cross-product

binary *relations*, that is binary relations of the form $A \times B$. Thus Keenan (1992) proved the following theorem which can be used to show that some functions are not Fregean (see also van Eijck (2005)):

Proposition 1. *(Keenan) If F_1 and F_2 are Fregean (type $\langle 2 \rangle$) quantifiers then $F_1 = F_2$ iff for all $A, B \subseteq E$ it holds that $F_1(A \times B) = F_2(A \times B)$*

I am interested in the reducibility type $\langle 2 \rangle$ quantifiers induced in some way by subject NPs and by expressions "containing" reflexives. The following definition makes this more precise:

Definition 5. Let $\langle NP, GNP \rangle$ be a sequence such that NP denotes the type $\langle 1 \rangle$ quantifier Q and GNP denotes the type $\langle 2:1 \rangle$ function F. Then the sequence $\langle NP, GNP \rangle$ induces a type $\langle 2 \rangle$ quantifier G iff $G(R) = Q(F(R))$.

I will also say, somewhat ambiguously, that in this case the sequence $\langle Q, F \rangle$ induces the quantifier G or that the GNP (or its denotation, the function F) induces the type $\langle 2 \rangle$ quantifier.

Let me consider now some correlations between the properties of NP and GNP (or of their denotations) and the reducibility of the quantifier they induce. Obviously, if the GNP is an NP then the sequence $\langle NP, GNP \rangle$ induces a Fregean quantifier. Interestingly not only GNPs which are NPs can induce Fregean quantifiers. This is the case when the NP in the sequence $\langle NP, GNP \rangle$ is a proper name and the GNP is, roughly speaking, a simple or complex reflexive expression. Proper names denote individuals, that is ultrafilters generated by the element of the universe E which is the referent of the proper name. More precisely if the PrN refers to a ($a \in E$), then PrN denotes the individual I_a defined as follows:

Definition 6. $I_a = \{X : X \subseteq E \wedge a \in X\}$.

The following proposition shows that the sequence $\langle PrN, GNP \rangle$, where GNP denotes a predicate invariant function, always induces a Fregean quantifier:

Proposition 2. *Let F be a type $\langle 2:1 \rangle$ predicate invariant function. Define a type $\langle 2 \rangle$ quantifier G_{F,I_a} as follows: $G_{F,I_a}(R) = 1$ iff $I_a(F(R)) = 1$. Then G_{F,I_a} is Fregean for any $a \in E$.*

Proof. Define the function h_F which maps every $a \in E$ to a type $\langle 1 \rangle$ quantifier in the following way: $h_F(a)(Y) = 1$ iff $a \in F(\{a\} \times Y)$. Since F is predicate invariant we have $y \in F(R)$ iff $y \in F(\{y\} \times yR)$ (because $yR = y(\{y\} \times yR)$). From this it follows that $G_{F,I_a}(R) = I_a((h_F(a))_{acc}(R)))$ for any $a \in E$. Thus G_{F,I_a} is equivalent to $Q_1(Q_{acc})$ where $Q_1 = I_a$ and $Q = h_F(a)$. □

Thus a proper name and a GNP which denotes a predicate invariant function always induce Fregean quantifiers. This means that the type $\langle 2 \rangle$ quantifiers involved in the interpretation (1) above are Fregean. Similarly with the quantifier involved in (5):

(5) Al shaved nobody but himself and Leo.

Using proposition 1 it is easy to show that type $\langle 2 \rangle$ quantifiers involved in the interpretation of (2) above are not Fregean (Keenan (1992)). I present now a general result from which this fact follows. Consider:

Definition 7. The set of sets $Ft(C)$ called *filter generated by the set C* is defined as follows: $Ft(C) = \{X : X \subseteq E \wedge C \subseteq X\}$

Thus an ultrafilter is a filter generated by a singleton. However, not only proper names denote filters. For instance a conjunction of proper names denotes a filter (generated by the union of their referents). Similarly universally quantified NPs like *Every (student)* denote filters.

The following proposition is easy to prove:

Proposition 3. *Let $Q = Ft(C)$ for some $C \subseteq E$, $|C| \geq 2$. Then:*
(i) $Q(SELF(X \times Y) = Q(Q_{acc}(X \times Y)$, for any $X, Y \subseteq E$
(ii) $Q(SELF((C \times C) \cap Id)) \neq Q(Q_{acc}((C \times C) \cap Id))$, where $Id = \{\langle x,x\rangle : x \in E\}$

Observe that the relation $(C \times C) \cap Id$ is not a cross-product relation. Thus it follows from proposition 3 that the quantifier in (2) above is not Fregean.

In fact more can be shown. The following proposition is a consequence of proposition 3 and of the properties relating Boolean operations on sets to Boolean propositional connectives:

Proposition 4. *Let $Q = Ft(C)$ and $F = SELF \oplus (Q_1)_{acc}$, where $|C| \geq 2$, $\oplus$ is a Boolean connector and Q_1 a type $\langle 1\rangle$ quantifier. Then the following holds:*
(i) $Q(F(X \times Y)) = Q((Q \oplus Q_1)_{acc}(X \times Y))$, for any $X, Y \subseteq E$
(ii) $Q(F((C \times C) \cap Id)) \neq Q((Q \oplus Q_1)_{acc}((C \times C) \cap Id))$

As an example consider (6):

(6) Leo, Lea and every philosopher hate themselves and most logicians.

It follows from proposition 4, given that the meet of two filters is a filter, that the quantifier induced by the NP *Leo, Lea and every philosopher* and by the GNP *themselves and most logicians* (as they occur in (6) below) is not Fregean.

2 Strongly irreducible quantifiers

The proof of irreducibility of quantifiers induced by a type $\langle 1\rangle$ quantifier and the function *SELF* discussed in the previous section essentially involves proposition 1. In order to decide whether quantifier F is (Frege) reducible two steps are necessary. First, a Fregean quantifier G which takes the same values as F on cross-product relations has to be found. Second, one has to show that both quantifiers differ on a non-cross-product relation. Neither of these steps is obvious. To construct the quantifier G when the quantifier F is induced the sequence $\langle Ft(C), SELF\rangle$ is easy: it is enough, as we have seen, to replace *SELF* by $Ft(C)_{acc}$. This move does not apply, however, to other cases, when the NP does not denote a filter, and in particular not to the case illustrated in (3) above.

Indeed, it has been observed by Ben Shalom (1994) that (7) is not equivalent on cross product-relations to (8a) but rather to (8b). Similarly (9a) is not equivalent to (9b):

(7) Two students criticised themselves.

(8) a. Two students criticised two students.

b. Two students criticised the same two students.

(9) a. Only Leo shaved himself.
b. Only Leo shaved Leo.

Ben Shalom (1994) proposes another way of solving the problem of reducibility of quantifiers in general and shows in particular that the type $\langle 2 \rangle$ quantifier involved in the interpretation of (7) is not Fregean. Still another solution to the above problem is offered by van Eijck (2005) who proves the following:

Proposition 5. *(van Eijck) : Let F be a type $\langle 2 \rangle$ quantifier such that $F(\emptyset) = 0$. The reduct F^* of F is defined as follows: $F^* = Q_1(Q_2)_{acc}$ where Q_1 and Q_2 are positive type $\langle 1 \rangle$ quantifiers such that $Q_1(X \neq \emptyset) = 1$ iff $\exists_{B \subseteq E} F(X \times B) = 1$ and $Q_2(Y \neq \emptyset) = 1$ iff $\exists_{A \subseteq E} F(A \times Y) = 1$. Then F is Fregean iff $F^* = F$.*

It is not difficult to show that if Q is positive (that is if $Q(\emptyset) = 0$) and $F(R) = Q(SELF(R))$ then $F^*(R) = Q(Q_{acc}(R))$. From this fact the Frege non-reducibility of many quantifiers follows in particular the non-reducibility of the quantifier given in the example (3) above.

In (3) the subject NP *Only Leo* denotes an atomic $\langle 1 \rangle$ quantifier (an atom of the algebra of type $\langle 1 \rangle$ quantifiers). This is a quantifier which contains just one element. More precisely, for any $A \subseteq E$ the quantifier Q_A is atomic iff $Q_A(X) = 1$ iff $X = A$. The following property holds for "most" atomic quantifiers:

Proposition 6. *Let Q_A be atomic quantifier having just A as its only element such that $\emptyset \neq A \neq E$. Then Q_A is neither a union of individuals, nor an intersection of individuals nor a finite symmetric difference of individuals (where the symmetric difference of two sets X and Y = $(X \cap Y') \cup (Y \cap X')$)*

The above proposition and the following proposition 7 proved by Westerståhl (1996) will be used to show that atomic type $\langle 1 \rangle$ quantifiers and the function $SELF$ induce non-Fregean quantifiers:

Proposition 7. *Let Q be a positive type $\langle 1 \rangle$ quantifier (that is $Q(\emptyset) = 0$). Then the equality $Q(Q_{acc}(R)) = Q(Q_{acc}(R^{-1}))$ holds iff Q is either a union or intersection of individuals or a finite symmetric difference of individuals.*

Consider now a type $\langle 2 \rangle$ quantifier F defined as $F(R) = Q(SELF(R))$, where Q is an atomic type $\langle 1 \rangle$ quantifier containing A as only element and such that A is not empty and not equal to E. Observe first that F is convertible, that is $F(R) = F(R^{-1})$. Furthermore, the reduct F^* of F has the following form: $F^* = Q(Q_{acc})$. It follows from propositions 6 and 7 that $F^* \neq F$ and thus the quantifier $F = Q(SELF)$ is not Fregean.

The above discussion about the use of proposition 1 in demonstrating the irreducibility of some quantifiers shows that it might be interesting, not only for theoretical reasons, to distinguish the reducibility of quantifiers "detectable" by proposition 1 and other types of reducibility. More precisely (Zuber (2003)):

Definition 8. A type $\langle 2 \rangle$ quantifier F is *weakly reducible* iff there exists two type $\langle 1 \rangle$ quantifiers Q and Q_1 such that $F(X \times Y) = Q_1(Q_{acc}(X \times Y))$, for any $X, Y \subseteq E$,

A quantifier which is not weakly reducible is strongly irreducible (strongly non-Fregean). Obviously Fregean quantifiers are weakly reducible. There are, however, non-Fregean quantifiers which are weakly reducible. In fact we have already seen such quantifiers in connection with propositions 3 and 4: quantifiers induced induced by a filter (generated by a non-singleton) and the function *SELF* are such quantifiers. As a more abstract example, consider the atomic type $\langle 2 \rangle$ quantifier $F_{A\times B}$ which is true of just the relation $A \times B$, for $A, B \neq \emptyset$ and $A \neq E$. This quantifier is not-Fregean, because, as Keenan (1992) shows, atomic quantifiers are Fregean just in case the only relation of which they are true is of the form $E \times X$. Furthermore, it is easy to see that $F_{A\times B}(X \times Y) = Q_A((Q_B)_{acc}(X \times Y))$, where Q_A and Q_B are atomic type $\langle 1 \rangle$ quantifiers.

To show that a quantifier is not weakly reducible, the following proposition can be used Zuber (2003):

Proposition 8. *A type $\langle 2 \rangle$ quantifier is strongly irreducible iff for some sets P, P_1, P_2, P_3, P_4, $S_1$and S_2, the following holds: $F(P_1 \times S_1) \neq F(P_2 \times S_1)$, $F(P_3 \times S_2) \neq F(P_4 \times S_2)$ and $F(P \times S_1) \neq F(P \times S_2)$.*

Using proposition 8 it is easy to show that any sequence $\langle Q_A, SELF \rangle$, where Q_A is an atomic quantifier, induces a strongly irreducible quantifier. Consequently the quantifier induced by $\langle$*only Leo, himself*$\rangle$ (as found in (3)) is strongly irreducible.

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