UCLA Working Papers in Linguistics, Theories of Everything
Volume 17, Article : - , 2012

A note on invariance of grammatical categories

Denis Paperno

This squib aims to further our understanding of the relation between invariants of grammar and grammatical categories. Keenan and Stabler (2003) propose to formalize of the notion of 'structural/ grammatical' in terms of automorphism invariance, based on a very general notion of formal grammars (Bare Grammars). It is natural to think that category labels (more precisely: sets of language expressions that bear a given category label) are flesh and blood of grammatical structure and should be invariant. Indeed, categories are invariant (under stable automorphisms) in all of Keenan and Stabler's example grammars. But in fact it is possible to construe several types of counterexamples where grammatical categories fail to be invariant. I conjecture that automorphism invariance characterizes only categories whose content is properly syntactic, excluding e.g. morphological properties, which are traditionally also considered grammatical.

Keywords Bare Grammar, invariants, grammatical categories

Background

Keenan and Stabler (2003) propose a neat formalization of the linguistic notion of *structural* in terms of automorphism invariance. They rely on the a simple grammar format, whereby a grammar G is a quadruple

$$\langle \mathrm{V_G}, \mathrm{Cat_G}, \mathrm{Lex_G}, \mathrm{Rule_G} \rangle$$

where $\mathrm{V_G}$ and $\mathrm{Cat_G}$ are sets of vocabulary items and category symbols, respectively. Possible expressions have the form (s,C) where s$\in \mathrm{V}^*_\mathrm{G}$ and C$\in \mathrm{Cat_G}$. The set of lexical items $\mathrm{Lex_G}$ is a subset of $\mathrm{V_G} \times \mathrm{Cat_G}$, and each rule $R \in \mathrm{Rule_G}$ is a partial function from sets of expressions to expressions (from $(\mathrm{V}^*_\mathrm{G} \times \mathrm{Cat_G})^+$ into $\mathrm{V}^*_\mathrm{G} \times \mathrm{Cat_G}$).

The language $\mathrm{L_G}$ generated by grammar G is defined as the closure of $\mathrm{Lex_G}$ under $\mathrm{Rule_G}$:

$$\mathrm{Lex}_0 = \mathrm{Lex_G};$$

$$\mathrm{Lex}_{n+1} = \mathrm{Lex}_n \cup \{R(e_1, \ldots, e_k) \mid R \in \mathrm{Rule_G}, e_i \in \mathrm{Lex}_n\};$$

$$\mathrm{L_G} = \bigcup_{n \in \mathbb{N}} \mathrm{Lex}_n$$

A map h from $\mathrm{L_G}$ to $\mathrm{L_G}$ can be extended to a map h^* on relations on $\mathrm{L_G}$ so that $h^*(R) = \{h(e_1), \ldots, h(e_k) \mid e_1 \ldots e_k \in R\}$. In particular, h^* applies to functions on L^*_G and to the subsets of L_G, which can be seen, standardly, as special cases of relations. An *automorphism* on L_G is a map h that fixes Rule_G, i.e. for each $R \in \text{Rule}_G$, $h^*(R) = R$ (recall that rules are functions on L_G^*). Using this notion of automorphism (a map from language onto itself that fixes rules of grammar), (Keenan and Stabler 2003:21) conjecture:

> The *syntactic invariants* of a grammar G are the fixed points of the automorphisms of G.

So syntactic invariants are expressions, sets of expressions or relations on expressions that any automorphism on the language maps to themselves. Provably, the relations CON 'be constituent of', CC 'c-command' are invariant in all grammars; the grammatical formatives are "lexical items that are always mapped to themselves by the structure preserving transformations [= automorphisms – D.P.] on L_G." Keenan and Stabler argue that the anaphor-antecedent relation, theta role equivalence, grammatical case, and other syntactically relevant properties of and relations on natural language are invariant (fixed by all automorphisms of grammar). Most of the suggested invariance universals are quite plausible; but one, I claim, requires closer scrutiny.

Following the idea that structural equals invariant, Keenan and Stabler hypothesize that in all natural languages, syntactic categories are invariant (p. 23):

> For all $C \in \text{Cat}_G$, PH(C) [the set of expressions of category C – D.P.] is invariant (that is, the property of being a C is structural).

Later in the same work, Keenan and Stabler weaken this conjecture, requiring invariance only under *stable* automorphisms – those which can be extended properly under any extension of Lex_G (all the automorphisms considered below are stable). So (Keenan and Stabler 2010:13) hypothesize that

> for a given lexical item d, the other lexical items of the same category as d are just those $\langle \dots \rangle$ that a stable automorphism can map d to.

A related idea that (Keenan and Stabler 2010:9) entertain (and reject) equates syntactic categories with equivalence classes of the coarsest congruence, where a *congruence* is an equivalence relation $\simeq$ such that if $s_i \simeq t_i$ for all $\mathbb{N}$, then for any $R \in \text{Rule}$ and any $k \in \mathbb{N}$ either $R(s_1, \dots, s_k)$ and $R(t_1, \dots, t_k)$ are both undefined or $R(s_1, \dots, s_k) \simeq R(t_1, \dots, t_k)$. The grammars in the next section illustrate three types of counterexamples to both the original hypothesis (with or without the restriction to stable automorphisms) and the congruence-based hypothesis.

Example grammars

In fact, without additional assumptions the invariance of syntactic categories is not guaranteed (even just under stable automorphisms). A trivial case is category distinctions that play no role in the grammar. So for example one could assign distinct categories to animate and inanimate proper nouns of English:

$$\text{Lex}_{\text{Eng}} = \{(\text{Susan}, N_a), (\text{Leslie}, N_a), (\text{Martha}, N_a), (\text{Titanic}, N_i), (\text{Britain}, N_i), (\text{sank}, V)\}$$

$$\text{Rule}_{\text{Eng}} = \{\text{Merge}\}$$

where y=Merge((s,C),(t,C′)) iff C is N_a or N_i and C′=V and y=(s^t,S)

Given this grammar (which marks nouns for animacy but incorporates the idea that animacy plays no role in English) a map substituting an animate and an inanimate noun in all expressions would be an automorphism (a stable one):

$$h(\text{Susan}, N_a) = (\text{Britain}, N_i); h(\text{Britain}, N_i) = (\text{Susan}, N_a)$$

$$\text{and for all other } e \in \text{Lex}_\text{G}, h(e) = e.$$

Clearly, one of the categories N_a, N_i is redundant (in the following technical sense: one can replace all lexical items of category N_a with ones of N_i – and vice versa –without changing the string image of the language – $\{s \mid (s,C) \in \text{L}_\text{G} \text{ for some } C \in \text{Cat}_\text{G}\}$). Theoretical linguistics tends to avoid such redundancy of grammatical description, so the last example is hardly realistic for a grammar of natural language to be constructed by linguists. But redundant syntactic categories are merely a trivial special case of non-invariant ones – and more realistic examples are to follow.

So how can a category matter (i.e. be non-redundant in the grammar) without being structurally invariant? The short answer is that non-invariant category distinctions can encode the string operations on expressions but not their combinatory capabilities. Note that automorphisms have to fix rules, construed set-theoretically as relations on expressions: that $e_1, \ldots, e_k$ combine into e via rule R is, in Bare Grammar, formalized as $\langle e_1, \ldots, e_k, e\rangle \in R$ where R includes as a subset a k+1-ary relation (equivalently, in function notation, $R(e_1, \ldots, e_k) = e$). So combinatory properties encoded in categories have to be structural. E.g. in the English example above nouns can combine only with verbs and not with other nouns to produce a sentence; so any automorphism has to preserve the noun-verb distinction, otherwise the automorphism fails to fix the domain of Merge — and by implication Merge itself — and so is not an isomorphism, leading to a contradiction.

So a non-redundant category can be non-invariant if it matters only for the string component of what a rule does to the input expressions; domains and ranges of rules have to stay fixed. Linear order is one of the simplest examples of how the operation on strings can vary depending on the syntactic category of constituents combined. The category of an expression can indicate whether it is preposed or postposed when combining with another expression (this is standardly encoded in categorial grammar slash notation), and a single rule can serve both the preposed and the postposed case. Then if linear position is the only difference between two categories, expressions of these categories can be interchanged by automorphisms, so the categories are not invariant. Take the example of a language that has both prepositions and postpositions, assuming that both produce the same kind of adpositional phrases (PP), Ger with Rule$_\text{Ger}$={Merge}:

$$\text{Merge}((s,C),(t,C')) = \begin{cases} (s^\wedge t, PP) & \text{if } C = PP/N \text{ and } C' = N; \\ (t^\wedge s, PP) & \text{if } C = N\backslash PP \text{ and } C' = N; \\ \text{undefined} & \text{otherwise.} \end{cases}$$

$$\text{Lex}_\text{Ger} = \{(\text{Max}, N), (\text{Karl}, N), (\text{trotz}, PP/N), (\text{gemäß}, PP/N), (\text{zufolge}, N\backslash PP)\}$$

The example grammar above uses German words for illustration since German has both numerous prepositions (*trotz* 'despite', *gemäß* 'according to' etc.) and a few postpositions such as *zufolge* 'according to'. Take the map h that interchanges $(\text{gemäß}, PP/N)$ and $(\text{zufolge}, N\backslash PP)$, and also interchanges $(\text{gemäß Karl}, PP)$ with $(\text{Karl zufolge}, PP)$ and $(\text{gemäß Max}, PP)$ with $(\text{Max zufolge}, PP)$ but maps all other expressions to themselves. This map is an automorphism since it preserves Merge as can be easily seen on a case-by-case basis since the language is finite. The only non-trivial cases are expressions containing *zufolge* or *gemäß*, for instance

$$h(\text{Merge}((\text{gemäß}, PP/N)(\text{Karl}, N))) = h(\text{gemäß Karl}, PP) = (\text{Karl zufolge}, PP) =$$

$$= \text{Merge}((\text{zufolge}, N\backslash PP), (\text{Karl}, N)) = \text{Merge}(h(\text{gemäß}, PP/N), h(\text{Karl}, N))$$

So in this case, h commutes with Merge, our only rule here; other cases are analogous. This means that h fixes Rule_G and is by definition an automorphism, indeed a stable one (it naturally extends to new expressions of the language should we add new lexical items). But h fails to preserve categories PP/N and $N\backslash PP$, so these are not invariant (they are also non-redundant!). Therefore, prepositional and postpositional phrases turn out to be structurally isomorphic.

There is some linguistic evidence that in actual German, as opposed to our simplified example, prepositional and postpositional phrases are not isomorphic, but it has no bearing on the theoretical point made here: category distinctions that encode only the linear position of the expression with respect to others may fail to be invariant. Other than adpositions, some reasonable candidates for such category distinctions are preposed and postposed adjectives in French, phrase-initial vs. second-position elements in Indo-European languages (Latin *et* 'and' vs. second-position *-que* 'and'), and the preposed definite vs. postposed indefinite article of Classical Arabic.

Categories can affect not just linear order of constituents but also string operations applied to them by the same rule. A poster child could be inflection classes, as observed in numerous languages, compare the conjugation of Ancient Greek verbs in present indicative:

person	'say'	'release'
1sg	phē-mi	lu-ō
2sg	phē-s	lu-eis
3sg	phē-si	lu-ei

Inflection classes can be formalized as distinct categories in a grammar like the following:

$$\text{Rule}_{Gr} = \{\text{Form}_{1s}, \text{Form}_{2s}, \text{Form}_{3s}\}$$

$$\text{where Form}_{1s}(s, C) = \begin{cases} (s^\wedge\text{mi}, V) & \text{if } C = V_{mi} \\ (s^\wedge\text{ō}, V) & \text{if } C = V_o \\ \text{undefined} & \text{otherwise.} \end{cases}$$

$$\text{Form}_{2s}(s, C) = \begin{cases} (s^\wedge\text{s}, V) & \text{if } C = V_{mi} \\ (s^\wedge\text{eis}, V) & \text{if } C = V_o \\ \text{undefined} & \text{otherwise.} \end{cases}$$

$$\mathrm{Form}_{3s}(s,C) = \begin{cases} (s^\wedge \mathrm{si}, V) & \text{if } C = V_{mi} \\ (s^\wedge \mathrm{ei}, V) & \text{if } C = V_o \\ \text{undefined} & \text{otherwise.} \end{cases}$$

$$\mathrm{Lex}_{\mathrm{Gr}} = \{(\mathrm{lu}, V_o), (\mathrm{pher}, V_o), (\mathrm{leg}, V_o), (\mathrm{ph\bar{e}}, V_{mi}), (\mathrm{hist\bar{e}}, V_{mi})\}$$

Given this grammar, the map h that exchanges the stems and all forms of the verb (lu, V_o) and $(\mathrm{ph\bar{e}}, V_{mi})$ and maps all other expressions to themselves is a stable automorphism (as can be easily shown on a case-by-case basis). So inflection classes, even though they play an important role in Greek grammar (and are clearly not redundant), are not structural.

Conclusion

We identified two linguistic phenomena — positional classes and inflection classes — which are an indispensable part of the grammar of respective natural languages and can be encoded as grammatical categories, but may fail to be invariant under (stable) automorphisms. So positional and inflection classes are not syntactic invariants — and indeed, this makes informal sense. Inflection classes are traditionally treated in morphology rather than syntax, and it is a common assumption in modern generative grammar that linear order (as opposed to the phrase structure underlying it) plays no role in the syntax. Many linguists would agree that the counterexamples to invariance of categories should be formalized in different components of grammar than syntax proper: inflection classes within a dedicated morpho(phono)logical component, and the alternations in linear position “in a special component devoted to cliticization and readjustment” (Zwicky and Pullum 1983:385). To summarize, I conclude that categories of Bare Grammars are invariant to the extent that the content of those categories pertains to (narrow) syntax. Category distinctions that are purely morphological (the Greek conjugation example), linear (the German adposition case), or semantic (the English animacy distinction), do not need to be syntactic invariants.

References

Keenan, Edward L., and Edward P. Stabler. 2003. *Bare grammar: Lectures on invariants*. Stanford Monographs in Linguistics. CSLI.

Keenan, Edward L., and Edward P. Stabler. 2010. On language variation and linguistic invariants. In *Papers in mathematical linguistics 1*, ed. Thomas Graf, volume 15 of *UCLA Working Papers in Linguistics*, 1–17.

Zwicky, Arnold M., and Geoffrey K. Pullum. 1983. Phonology in syntax: The Somali optional agreement rule. *Natural Language and Linguistic Theory* 1:385–402.

Affiliation

Denis Paperno
Center for Mind/Brain Sciences
University of Trento
paperno@ucla.edu