

En deçà de la frontière de Frege?

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Introduction

We discuss French sentences such as (1a), the English counterpart of which, in (1b), is discussed in Keenan (1992):

- (1) a. Différents élèves ont répondu à différentes questions
b. Different pupils answered different questions

We propose that they have a wider range of interpretations than assumed in Keenan (1992), which bears on the question of the reducibility of functions denoted by paired *different* as above.

1 Reducibility

The discussion around this kind of sentence centers on the reducibility of some type 2 functions to the composition of two type 1 functions. What does this mean?

To think about this question, assume informally that a type 1 function is a function mapping a n dimensional space into an $n-1$ dimensional space, a type 2 function is a function mapping a n dimensional space into an $n-2$ dimensional space, and more generally:

a type p function is a function mapping an n dimensional space, $n \geq p$ into an $n-p$ dimensional space.

For example, to use geometric intuition (in ordinary space), the inside of a sphere is a three dimensional object. Intersecting it with a straight line going through it (yielding all the points of the line that are inside the sphere) is a type 2 operation, since the result is a one dimensional object.

Is the function “intersect with a straight line” in ordinary space reducible to the composition of two type 1 functions?

The answer is positive: a straight line can be seen as the intersection of two planes P1 and P2. Intersecting the sphere with the first plane P1 and then intersecting the result with the second plane P2 is going to yield exactly the same result as intersecting the sphere with the straight line. But, crucially, intersecting the sphere with a plane is a type 1

* A Ed, pour toutes ces années de conversations stimulantes and sheer fun, avec juste un brin de géométrie.

operation as it maps the inside of the sphere (a 3 dimensional object) to the points on the plane that are inside the sphere (a 2 dimensional object). And intersecting this 2 dimensional object with the other plan is also a type 1 operation as it will yield the points on the line inside the sphere (a 1 dimensional object).

In the sentence, *John admires Mary*, the function $F = (\text{John}, \text{Mary})$ maps the binary relation *admire* (which is a set of pairs, that is 2-dimensional) to true (or to false), if John admires Mary (or not). True (or false) is of dimension zero (it is a constant). So F is of type 2.

F is reducible as it can be seen as the composition of two functions: the function $M = (\text{Mary})$ which maps the binary relation *admire* (a set of pairs, 2-dimensional) to the set of people who admire Mary (a set of individuals, 1-dimensional); and the function $J = (\text{John})$ which maps a set of individuals (e.g. the set of people who admire Mary) to true – 0-dimensional – if its intersection with the set $\{\text{John}\}$ is not empty, (and to false otherwise).

In other words, we can write: $F(\text{admire}) = J [M [\text{admire}]]$.

Intuitively, if we think of the relation *admire* as a set of pairs (x,y) , a function of type 2 on *admire* will be reducible if, in the subset of pairs (p, q) it maps to true (or to false), the choice of a q does not depend on the choice of a particular p . In other words, there should be a p -independent way, a general rule, to pick the q associated with a given p .

Keenan (1992) convincingly shows that there are unreducible type 2 functions in English (and by extension in some other languages) but we will suggest that this is too strong in the particular kind of case illustrated by sentence (1a).

According to Keenan (1992: 202), a sentence such as (1) would “mean (on its weakest reading) that there are at least two pupils and for all distinct pupils x, y , the questions that x answered are not exactly the same as the questions y answered”: accordingly, the iterated use of *different* guarantees (minimally) a one-to-one match between pupils and subsets of questions (as well as the existence of at least two pupils). Call this interpretation *Keenan’s weakest reading*. Thus to know which questions were answered, we need to know which pupil we are talking about: different pupils and different questions are not interpreted independently. The discontinuous (different pupils ... different questions) mapping the binary relation *admire* to true (or false) is thus an unreducible type 2 function.

Note that if Keenan’s weakest reading is indeed the weakest, sentence 1 could in principle be true if there were 3 pupils and 2 questions in total, but not if there were 4 pupils and 2 questions: this is because the power set of a set of cardinality 2 is $2^2=4$; since each student answered at least one question, there should be 4 non empty distinct subsets of questions to distinguish the 4 students which is not the case.

Although we will not elaborate here, one reason that sentences such as in (1) is significant is that unlike many (all?) other cases of unreducible type 2 functions, this case, if it is indeed unreducible, can’t (at least can’t simply) be handled in terms of ordinary scope and binding relations: the grammatical descriptive apparatus must be enriched beyond (the equivalent of) variable binding.

We will, for French at least, disagree with Keenan (1992) regarding what the weakest reading of (1a) is. In fact we argue that the meaning of (1a) is much less specified, allowing all sorts of interpretations one of which is Keenan’s weakest reading. We only

discuss French, but we believe (from informal surveys) that this holds of English too.

2 Plurals

First let us make a detour and look at the following French sentence:

- (2) a. Ils ont dit qu'ils étaient malades
b. They said that they were sick

It can have the following range of interpretations:

- (i) each man said that he himself was sick, or
(ii) each man said that all the men were sick, or
(iii) each man said that all the other men were sick, or
(iv) each man said that some other men were sick or
(v) each man said that some set of men (including himself or not)

In other words, (2) can be read as e.g. object distributive reflexive as in (i), object non distributive as in (iii), strong or weaker reciprocal as in (ii) or (iv) and many intermediate situations as in e.g. (v). And different contexts can make certain interpretations more salient than others. The following sentence :

- (3) a. Ils ont promis de ne pas être à leurs enterrements
b. They promised not to be at their burials

makes the reading corresponding to (iii) most salient, that corresponding to (i) a joke and that corresponding to (v) not easily accessible.

This behavior seems general when plurals are involved. Thus, the same type of ambiguity can be found in what is sometimes described as ambiguous reflexive/reciprocal constructions in French:

- (4) a. Les enfants se regardent (disons, dans un miroir)
b. The children are looking at themselves (say, in a mirror)

Such a sentence can also be read as e.g. object distributive reflexive as in (i), object non distributive as in (iii), strong or weaker reciprocal as in (ii) or (iv) and many intermediate situations as in e.g. (v) :

- (i) each child is looking at himself, or
(ii) each child is looking at all of the other children, or
(iii) each child is looking at the whole group of children, or
(iv) each child is looking at some other children, or
(v) each child is looking at a set of some children (including himself or not).

If we change the verb to *jalouser* (feel jealous towards)

- (5) a. Les enfants se jalouent
b. The children are feeling jealous of themselves/each other

The interpretations corresponding to (ii) or (iii) above clearly remain. That corresponding to (i) seems unavailable, while judgments are less clear for (iv) and (v). The unavailable or non salient interpretations have in common that the denotation of the object overlaps with that of the subject, in other words, there is a subject/object disjoint reference effect.

How should this be coded? The only difference between (4a) and (5a) is a lexical one (*jalouser* vs. *regarder*) and *jalouser* is not intrinsically (conceptually) antireflexive, although it is unusual to be jealous of oneself. This suggests that which interpretation is rejected is more a matter of pragmatics than of literal meaning. If true, we should be able to set up a context, linguistically or otherwise allowing the seemingly unavailable interpretations. An indeed, it is quite possible for the subject and object of *jalouser* or of *jealous* to be coreferential, as e.g. in the sentences below, which can be read like (i) above:

- (6) a. Comment peuvent-ils se jalouser (eux-mêmes)? Est-ce une marque de folie ?
b. How can they be jealous of themselves? Is this a symptom of craziness?

We conclude that such constructions as (4a) or (5a) are not ambiguous between reflexive and reciprocal interpretations. They involve two coreferent plurals (say the subject and *se*) and yield the many interpretations the co-occurrence of two plurals allow.

Following Beck (2000), we can model what is happening in terms of cover. Under such an approach, the difference between these interpretations is due to the availability of different covers where a cover is a set of subsets of a set *S*, the union of which subsets yields *S* (see Schwarzschild, 1996, for more details). Indeed many possible covers of the set of children are available, some being pragmatically selected based on salience: depending on what is relevant in the context, the children can be divided into different kinds of subsets (the classification can be based on different criteria, e.g. the age of children, their heights, their clothes, or along the dimension of the children themselves {children looked at by one child, children looked at by another child, etc...}), or indeed totally arbitrary covers corresponding e.g. to a particular perceived situation as children looking at themselves in some arbitrary way in a mirror (one looking at himself, another looking at two other children, a third at himself and another child, etc...).

3 Différent ... différent...

Let us now come back to (the French version of) (1). Consider first such sentences as below, each with only one instance of *différent*, in French in (7a), its English counterpart in (7b), and paraphrases in (7c) and (7d):¹

- (7) a. Différents élèves ont répondu
b. Different pupils answered
c. Pupils who are different from each other answered
d. A variety of pupils answered

¹ In French, the adjective *différent* could also be post nominal (with a plural indefinite article: *des étudiants différents*) with the same reading. However, a post nominal *différent* would also allow so-called external readings in which the pupils, say, are different from some entity introduced in the discourse. External readings are not relevant here, but see Charnavel (2012) for recent discussion.

- (8) a. J'ai répondu à différentes questions
 b. I answered different questions
 c. I answered questions which were different from each other
 d. I answered a variety of questions

These are perfectly fine sentences, e.g. appropriate (although not very informative) answers to questions such as Who answered? or What happened at your interview? respectively. Given these paraphrases we would expect sentence (9a) (=1a) with two instances of *différent* to have the meaning indicated in (9b) or (9c):

- (9) a. Différents élèves ont répondu à différentes questions
 b. Pupils different from each other answered questions different from each other
 c. A variety of pupils answered a variety of questions

Even though it may not be the most immediately salient reading, such a meaning is without question available. In fact it is difficult to see how this meaning could be blocked as it is one arising from the normal, compositional interpretive rules.

Given that such a meaning is available, where the subject and the object are interpreted independently of each other, and given that both the subject and the object are plurals (there should be at least two pupils - as Keenan notes - and two questions, these properties possibly being implicatures, see Spector, 2006) this leads, as we just saw, to many different interpretations depending on the covers chosen.

What would correspond to Keenan's weakest reading, is the choice of a cover along the dimension of the pupils {questions answered by one pupil, questions answered by another pupil, etc..} just in case no two pupils answered exactly the same questions.

But this is by no means the only cover available. Weaker (and also stronger) interpretations than Keenan's weakest reading seem to us available in ordinary usage for such sentences. For example, in a context in which there are 17 pupils and 4 questions, sentence (9a) seems to us to be an appropriate answer to the question "who answered what" even if two pupils answered exactly all the same questions. In fact, it seems fine (although perhaps not as informative as one could wish on the part of the speaker) if any number of students answered exactly all the same questions including the case in which all of them answered exactly the same questions.

Conclusion

If this is right, the (different... different) type 2 function is reducible as there is no semantic dependency between what questions are answered and who answers them. If we are right, such functions as (*différent* .. *différent* ...) in fact lie "en deçà" of the Frege boundary.

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