

## The Puzzle(s) of Absolute Adjectives On Vagueness, Comparison, and the Origin of Scale Structure

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This paper provides a unified solution to two puzzles involving vagueness and scalar adjectives of the *absolute* lexical class (ex. *bald*, *empty*, *straight* etc.). The first puzzle concerns the proper analysis of the relationship between contextual effects associated with absolute adjectives and those associated with *relative* adjectives like *tall*, *long* or *expensive*. The dominant view in philosophy is that *tall* and *bald* are both vague constituents. However, an emerging view in linguistics is that the context-sensitivity exhibited by *bald* is not due to vagueness but a different phenomenon: *imprecision* (cf. Kennedy (2007)). I argue that the properties of both *tall* and *bald* that appear to challenge our classical semantic theories are symptoms of a single phenomenon: vagueness. I provide a unified analysis of these properties that accounts for Kennedy's data using Cobreros, Égré, Ripley, and van Rooij (2010)'s *Tolerant, Classical, Strict* (TCS) non-classical logical framework. The second puzzle is the puzzle of the gradability of absolute adjectives. It has been observed that these constituents seem to have a non-context-sensitive meaning. However, on the other hand, they are clearly gradable (*John is balder than Peter*). I show how, using TCS, we can preserve the empirical benefits of the proposal that these adjectives have absolute semantics, while constructing the pragmatic scales associated with them using methods in the same vein as van Benthem (1982) and van Rooij (2011a).

*Keywords* Vagueness, Imprecision, Comparatives, Adjectives, Paraconsistent Logic, Scale Structure, Delineation Semantics, Semi-orders

### 1 Introduction

This paper provides a unified solution to two puzzles involving vagueness and scalar adjectives of the *absolute* lexical class, some examples of which are shown in (1).

- (1) a. John is **bald**.  
b. This room is **empty**.  
c. The stick is **straight**.

The first puzzle concerns the proper analysis of the relationship between contextual effects associated with absolute adjectives and those associated with *relative* adjectives like *tall*, *long* or *expensive*. I will call it the puzzle of **contextual variation in the presence of vagueness**. Following many authors (ex. Keefe (2000), Fara (2000), Smith (2008), among others), I take vague language to be characterized by the presence of three (related) properties: *borderline cases* (objects for which it is difficult or even impossible to tell whether they satisfy the predicate), *fuzzy boundaries* (the observation that there appear to be no sharp boundaries between cases of a vague predicate and its negation), and *susceptibility to the Sorites paradox* (a paradox for systems based on classical first order logic that follows from the fuzzy boundaries property). These properties will be discussed at great length throughout the paper.

As will be outlined below, relative adjectives are uncontroversial examples of vague constituents; however, as we will see, in some (or indeed most) contexts, the adjectives in (1) also seem to display the symptoms of vagueness. An open debate has emerged in the linguistic and philosophical literatures as to whether the appearance of the characterizing properties of vague language with absolute adjectives should be analyzed in a parallel manner to their appearance with relative adjectives. The dominant view in philosophy, both historically and recently, is that *tall* and *bald* are both vague constituents (cf. Fine (1975), Keefe (2000), Fara (2000), Smith (2008), Égré and Klinedinst (2011) among many others). However, an emerging (if not already standard) view in linguistics is that the context-sensitivity exhibited by *bald* is not due to vagueness but a different phenomenon: *imprecision* (cf. Pinkal (1995), Kennedy and McNally (2005), Kennedy (2007), Moryzcki (2011) and Husband (2011), among others, see also Sauerland and Stateva (2007) for a similar view).

My main proposal for the first part of the paper is that the traditional philosophical view is correct: I argue that the properties of both relative adjectives like *tall* and absolute adjectives like *bald* that appear to challenge our classical semantic theories are symptoms of a single phenomenon: vagueness. However, at the same time, I argue that the empirical observations made by Pinkal, Kennedy and others pose a serious challenge for those who wish to treat the vagueness of relative and absolute adjectives in a uniform manner. Therefore, one of the contributions of this paper is to provide a unified analysis of the puzzling properties of vague language in the adjectival domain that accounts for Kennedy's data.

In providing such an analysis, I will take a stand on another open debate in the literature that cross-cuts the vagueness vs imprecision debate. That is, regardless of how they treat *bald* and *empty*, authors differ on whether they think vagueness associated with *tall* is semantic or pragmatic. Those advocating a semantic view propose that the puzzling properties of vague language arise from aspects of the semantic denotation of scalar constituents. Thus, in this view, our classical semantic theories are ill-equipped to analyze vague predicates, and we need some alternative theory. Advocates of this view are *S<sup>v</sup>valuationists* (those who adopt *Supervaluationism* (ex. Fine (1975), Kamp (1975), Keefe (2000), among many others), and *Subvaluationism* (Hyde 1997)), those favouring fuzzy logical approaches (ex. Lakoff (1973)), and, indeed, proponents of most non-classical approaches to vague language. Those advocating the pragmatic view propose that scalar constituents

have classical semantic denotations and the problematic properties arise as a result of some context-dependent computation that takes these semantic denotations as input<sup>1</sup>. Advocates of this view include *Epistemicists* (ex. Williamson (1994) and others), *Contextualists* (ex. Kamp (1981), Soames (1999), Fara (2000), Shapiro (2006) among others) and also some authors that adopt non-classical logical approaches. I will show that a simple and empirically adequate unified analysis of the vagueness of relative and absolute adjectives is possible if we take vagueness to be a pragmatic phenomenon. In particular, I give an analysis in which scalar adjectives have precise semantic denotations which can be framed as in *Delineation Semantics* (Klein (1980) and others), and the characteristic features of vague language are the result of contextually dependent computation on these denotations. The analysis is framed within Cobreros et al. (2010)'s *Tolerant, Classical, Strict* (TCS) non-classical logical framework for modelling the properties of vague language.

Furthermore, this proposal has some important methodological implications for the way in which we should investigate vagueness in natural language. I argue that we can get a better handle on the complete range of data relevant to the study of vague constituents if we are explicit about the pragmatic component involved when we formulate our descriptive empirical generalizations. In particular, I argue that, contrary to the common use in the literature, the use of the term *vague* should be relativized to a linguistic or extra-linguistic context. In other words, *vague* should be used as a stage-level predicate: we should describe the data using language like (2-a), rather than the usual (2-b).

- (2) a. **Stage Level:** An adjective *P* is vague **in context c** iff. . .
- b. **Individual Level:** The adjective *P* is vague.

The individual-level use of *vague* is the heritage of the long tradition that presupposes a semantic analysis of the puzzling properties of vague language; however, I believe that this use has served to obscure both some important similarities and important differences between the relative and absolute classes of adjectives.

The second puzzle to be solved is **the puzzle of the gradability of absolute adjectives**. It has been long observed that, on the one hand, absolute adjectives seem to have a precise non-context-sensitive meaning (hence the term “absolute” adjective). However, on the other hand, as shown by the unexceptional examples in (3), they are clearly gradable and allow for some individuals to have a higher degree of “baldness” or “emptiness” etc. than others.

- (3) a. John is balder than Fred.
- b. This room is emptier than that one.
- c. This stick is straighter than that stick over there.

The most common response to this puzzle in the literature is to treat absolute adjectives as context-sensitive and gradable, just like relative adjectives, and then to

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<sup>1</sup>This way of cutting the *semantics/pragmatics* distinction is consistent with both the traditional characterization of pragmatics as the aspects of meaning that are context-dependent (as adopted in, for example, Montague (1968)) and the more recent characterization (by, for example, Stanley (2000)) as those context-dependent aspects of the computation of meaning that apply to semantic objects.

have an additional proposal for why they sometimes appear context-independent (ex. [Rotstein and Winter \(2004\)](#), [Kennedy \(2007\)](#), [Toledo and Sassoon \(2011\)](#), among others). However, I argue that these additional proposals are often ad hoc and miss the essence of what makes the absolute class different from the relative class. Instead, I take a different approach (more in line with [Kennedy and McNally \(2005\)](#) and [Récanati \(2010\)](#)) in which members of the absolute class have non-context sensitive denotations. I show how, using the TCS framework for modelling vague language, we preserve the intuition (and the empirical benefits) of the proposal that these adjectives have absolute semantics, while, at the same time, we can construct the pragmatic scales associated with absolute adjectives using methods in the same vein as [van Benthem \(1982\)](#) and [van Rooij \(2011a\)](#). I argue that my style of analysis is preferable because it opens the door to a more general theory of which kinds of semantic objects are susceptible to the phenomenon of vagueness.

The paper is organized as follows: In section 2, I present the (vagueness independent) syntactic and semantic arguments for the existence of a lexical class distinction between *relative* and *absolute* adjectives. Then, in section 3, I present the first puzzle (contextual variation in the presence of vagueness) in greater detail, and, in section 4, I present the second puzzle (the gradability of absolute adjectives). In section 5, I present a solution to puzzle 1: I give a unified analysis of the vagueness associated with *tall* and *bald* within the TCS framework. Then, in section 6, I present a solution to puzzle 2: I give a new analysis of the comparative construction formed with absolute predicates, and in section 7, I extend this analysis to relative comparatives. Finally, in section 8, I argue that the analysis proposed in sections 7 and 8 can predict not only results concerning the potential vagueness and gradability of scalar adjectives, but also predicts the non-vagueness and non-gradability of non-scalar adjectives, and thus we develop a new perspective on the origin of these lexical class distinctions in the adjectival domain.

I therefore conclude that, by adopting a Klein-ian approach to the semantics of gradable expressions and a similarity-based non-classical logical approach to their pragmatics, we can arrive at a more empirically adequate and more elegant analysis of an important class of vague and precise constituents in the adjectival domain.

## 2 The Relative/Absolute Distinction

It has been long observed that the syntactic category of bare adjective phrases (APs) can be divided into two principle classes: *scalar* (or *gradable*) vs *non-scalar* (*non-gradable*). The principle test for scalarity of an adjective *P* is the possibility of *P* to appear (without coercion) in the explicit comparative construction. Thus, we find a first distinction between adjectives like *tall*, *expensive*, *bald*, and *empty* (4) on the one hand and *atomic*, *pregnant* and *geographical* on the other (5).

- (4)
- a. John is taller than Phil.
  - b. This watch is more expensive than that watch.
  - c. John is balder than Phil.
  - d. My cup is emptier than your cup.

- (5) a. ?This algebra is more atomic than that one.  
b. ?Mary is more pregnant than Sue.  
c. ?This map is more geographical than that one.

In this section, I present the arguments from the literature in favour of the further division of the class of scalar adjectives into two lexical classes: what are often called the *relative* class and the *absolute* class. In particular, (following others) I show that, in languages like English, adjectives like *tall* and *expensive* pattern differently from ones like *bald* and *empty* with respect to a variety of independent syntactic and semantic tests. The tests that I present in this section are only a subset of the tests described in the literature, and the reader is encouraged to consult works such as Cruse (1986), Yoon (1996), Rotstein and Winter (2004), Kennedy and McNally (2005), and Kennedy (2007) for more information.

In Kennedy (2007), it is argued that another distinguishing feature of the two classes involves the availability of the symptoms of vagueness. However, I turn to this proposal in section 3. The point of the present section is to show that, regardless of whether or not we think that the absolute/relative distinction is relevant for vagueness, this is a lexical class distinction that has reliable non-vagueness-related grammatical effects, and therefore any theory of the semantics and pragmatics of scalar adjectives must propose a difference between *tall* and *bald* at some level.

## 2.1 Syntactic Criteria

I first show that absolute adjectives and relative adjectives behave differently with respect to the syntactic constructions that they are licensed in. Thus, at the very least, the absolute/relative distinction corresponds to a syntactic distinction. This observation has been made by many authors, and, by far, the most commonly studied syntactic differences between adjectives like *tall* and *bald* involve the distribution of adjectival modifiers. For example, relative adjectives are generally incompatible with *almost*<sup>2</sup>, while absolute adjectives are perfectly fine.

- (6) a. John is **almost** bald.  
b. The room is **almost** empty.
- (7) a. ?John is **almost** tall.  
b. ?This watch is **almost** expensive.

Other modifiers that distinguish *tall*, *expensive* etc. from *bald*, *empty* etc. are *completely* (8) and *absolutely* (9).

- (8) a. John is **completely** bald.  
b. ?John is **completely** tall.

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<sup>2</sup>Of course, it is sometimes possible to coerce some meaning from an absolute modifier paired with a relative adjective; however, the point is that there is a sharp contrast in naturalness between (6) and (7). See, in particular, Rotstein and Winter (2004) and Kennedy and McNally (2005) for discussion.

- (9) a. John is **absolutely** bald.  
 b. ?John is **absolutely** tall<sup>3</sup>.

As I mentioned, a major focus of the study the differences between relative and absolute adjectives has been the syntactic distribution of modifiers; however, the absolute/relative distinction has an effect on the felicity of many other syntactic constructions. For example, as noticed by Green (1972) and discussed in Wechsler (2005) and Beavers (2008) among others, only absolute adjectives are licensed in (one class) of resultative constructions in English:

- (10) Absolute adjectives  
 a. John wiped the table **clean/dry/smooth**.  
 b. John hammered the metal **flat/straight**.  
 c. John shook the box **empty**.
- (11) Relative adjectives  
 a. \*John wiped the table **expensive**<sup>4</sup>.  
 b. \*John hammered the metal **long/short**.

We can frame these generalizations concerning the class of relative adjectives (RA) and the class of absolute adjectives (AA) as follows<sup>5</sup>:

- (12) **Syntactic Criteria:**  
 If  $Q$  is an absolute adjective ( $Q \in AA$ ), then  $Q$  is licensed in the frames:  
 a. John is almost/completely/absolutely  $Q$ .  
 b. John  $V$ -ed the table  $Q$ . (for an appropriate resultative verb  $V$ )

In summary, there is ample empirical evidence that, independent of the vagueness/imprecision debate, languages like English treat adjectives like *tall* and *expensive* in a different way from *bald* and *empty* with respect to the types of syntactic constructions that these adjectives can appear in.

### 2.1.1 Partial Absolute Adjectives

The previous discussion was limited to comparing relative adjectives with what are sometimes called *total* or *universal* absolute adjectives (Kamp and Rossdeutscher (1994), Yoon (1996), and Rotstein and Winter (2004)). It is often argued that languages like English distinguish a third adjectival class, called the *partial* or *existential* adjectival class. This class contains adjectives like *wet*, *dangerous*, *sick* and *awake*.

It is uncontroversial that, from a syntactic point of view, the partial AAs behave

<sup>3</sup>Where *absolutely* modifies *tall* and not the degree of certainty of the speaker.

<sup>4</sup>Imagine that the price of the table increases with its cleanliness.

<sup>5</sup>Note that these syntactic tests actually classify colour terms like *red* as relative rather than absolute, as is sometimes proposed (ex. in De Swart (1998)).

- (i) a. ??This painting is **completely/absolutely red**.  
 (?? on the non-Q-float reading)  
 b. ??John wiped the table **red**.  
 (It was red underneath all the dirt.)

differently from the total AAs. As discussed in [Rotstein and Winter \(2004\)](#), one way in which partial AAs distinguish themselves from total AAs through the possibility of modification by *slightly* in English.

(13) **Partial Absolute Adjectives**

- a. This towel is slightly wet.
- b. John is slightly awake.
- c. This ride is slightly dangerous.
- d. Peter is slightly sick.

(14) **Total Absolute Adjectives**

- a. ?This towel is slightly dry.
- b. ?John is slightly asleep.
- c. ?This ride is slightly safe.
- d. ?This room is slightly empty.

What is more controversial is whether partial adjectives show different syntactic behaviour from relative adjectives. As discussed in [Solt \(2011\)](#), relative adjectives are also compatible with *slightly*, where they receive a ‘slightly too’ interpretation.

(15) **Relative Adjectives**

- a. John is slightly tall (for his age).
- b. My hair has gotten slightly long.
- c. He’s slightly friendly.
- d. Don’t you think you’re slightly fat? ([Solt 2011](#))

Furthermore, partial AAs behave exactly like relative adjectives in that they disallow modification by *completely* (where the modifier applies to the adjective, not the parts of the subject), and are not possible in *hammer/wipe*-type resultatives.

- (16) a. \*This towel is completely wet.  
b. \*John wiped the table wet.

In fact, to my knowledge, there are no modificational constructions or other syntactic constructions that either pick out the partial adjectives as a class or group partial AAs together with total AAs to the exclusion of relative adjectives<sup>6</sup>. Thus, from a syntactic point of view, natural languages in (at least) the Indo-European family clearly distinguish *tall* from *bald*, and *bald* from *wet*; however, from this angle, it is not so clear to what extent they make a distinction between *tall* and *wet*.

## 2.2 Semantic Criteria

Another class of tests, semantic tests, distinguish adjectives like *tall* from those like *bald*. One such test is the accentuation test. As discussed in [Kennedy \(2007\)](#)

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<sup>6</sup>One possibility might be *almost*: [Rotstein and Winter \(2004\)](#) (p. 9) give some examples with *almost sick* and *almost dangerous*. However, their examples do not seem to me to be any more natural than a coercion of *almost expensive*.

(p. 46) (based on Unger (1975)), accenting a total absolute adjective forces a precise interpretation, in which case increasing the degree to which the subject satisfies the predicate results in a contradiction.

- (17) a. My glass is FULL # but it could be fuller.  
b. This line is STRAIGHT # but you could make it straighter.

Accenting a relative adjective for reasons other than contrastive focus can seem strange; if anything, the predicates in (18) get an intensive interpretation ("really tall/interesting" or "definitely tall/interesting"). In any case, increasing the degree to which the relative predicate holds of the subject is perfectly fine.

- (18) a. This film is INTERESTING, but it could be more interesting.  
b. Sarah is TALL, but she could be taller if she wore heels.

In the accentuation test, partial adjectives pattern like relative adjectives: when accentuated, they get an intensive interpretation and are still compatible with the comparative. Again, we might wonder whether they really constitute a separate lexical class from RAs:

- (19) a. This towel is WET, but it could be wetter.  
b. This neighbourhood is DANGEROUS, but it could be more dangerous.

It is sometimes argued that partial adjectives must have a distinct analysis from relative adjectives like *tall* because elements of the former class give rise to certain entailments that are not present with those of the latter class. For example, at first glance, it appears that the following inferences from the comparative form to the positive form hold with partial adjectives, but not with relative adjectives.

- (20) **Partial Adjectives**  
a. The table is wetter than the chair.  $\Rightarrow$  The table is wet.  
b. This pair of pants is dirtier than that pair of pants.  $\Rightarrow$  This pair of pants is dirty.  
c. John is sicker than Mary.  $\Rightarrow$  John is sick.

- (21) **Relative Adjectives**  
a. John is taller than Mary.  $\not\Rightarrow$  John is tall.  
b. This stick is longer than that stick  $\not\Rightarrow$  This stick is long.  
c. This book is more expensive than that magazine  $\not\Rightarrow$  This book is expensive.

However, it is not clear that these inferences always go through. One example of where they can fail is with the partial adjective *dangerous*. For instance, statistically speaking, there are fewer casualties per kilometre travelled by plane than by car. So it is reasonable to say that driving from one city to another one is more dangerous than flying there. However, it seems inappropriate to conclude from this fact that driving from one city to another is itself dangerous in the absolute.



- (22) a. Driving from Ottawa to Toronto is more dangerous than flying from Ottawa to Toronto  $\nRightarrow$  Driving from Ottawa to Toronto is dangerous.  
b. Leaving the house at least once in one's life is more dangerous than never leaving.  $\nRightarrow$  Leaving the house at least once in one's life is dangerous.

Thus, with at least some partial adjectives, the threshold for satisfying the predicate is context-sensitive, like the threshold for satisfying a relative adjective<sup>7</sup>.

The thresholds for total AAs, on the other hand, do not seem to be context-sensitive (in the same way), and, thus, these constituents license negative inferences from the comparative to the positive:

- (23) **Total Absolute Adjectives:**  
a. This room is emptier than that room.  $\Rightarrow$  That room is not empty.  
b. John is balder than Phil  $\Rightarrow$  Phil is not bald.  
c. This road is straighter than that road.  $\Rightarrow$  That road is not straight.

Relative adjectives and partial adjectives again pattern together against total adjectives in not having these entailments.

- (24) a. **RA:** John is taller than Phil  $\nRightarrow$  Phil is not tall.  
b. **Partial AA:** This towel is wetter than that towel  $\nRightarrow$  That towel is not wet.

Thus, while we find that there are semantic tests that reliably distinguish total AAs from RAs and partial AAs, things are more complicated when it comes to distinguishing between RAs and partial AAs.

### 2.3 Summary

In summary, we find that relative adjectives (like those in (25)) consistently behave differently from total absolute adjectives (like those in (26)) in a variety of syntactic and semantic tests.

- (25) **Relative Adjectives:**  
tall, short, expensive, cheap, nice, friendly, intelligent, stupid, narrow, wide...  
(26) **Total Absolute Adjectives:**  
bald, empty, clean, smooth, dry, straight, full, flat, healthy, safe, closed...

Although they are classified as 'absolute' adjectives by Kennedy (2007), partial AAs (like those in (27)) behave like relative adjectives in the vast majority of the tests that we have seen in this section.

- (27) **Partial Absolute Adjectives:**  
dirty, bent, wet, curved, crooked, dangerous, awake, open...

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<sup>7</sup>This fact is acknowledged by many authors, including Rotstein and Winter (2004) and Kennedy (2007).

The summary of the data is shown in table 1.

	Test	Total AAs	Partial AAs	RAs
Syntactic	<i>completely/almost</i>	✓	×	×
	Resultatives	✓	×	×
Semantic	Accentuation $\perp$	✓	×	×
	Negative Comparative $\models$	✓	?	×

Table 1: (Sample of)The tests for the relative/absolute distinction

What table 1 shows is that, in all the tests discussed in this section, and, to my knowledge, all the tests discussed in the literature, partial adjectives either behave straightforwardly like relative adjectives and unlike total absolute adjectives, or they display some variation. I therefore suggest that, given these data, it is reasonable to suppose that grammars like English only make two principled lexical class distinctions: the **relative** class, which includes the elements in (25) and (27), and the **absolute** class, which includes the elements in (26). Therefore, throughout the rest of this paper, I will primarily focus on analyzing the differences between clear cases of RAs like (25) and clear cases of AAs like (26), leaving open the question of whether we should really be treating the adjectives in (27) differently from those in (25). Having knock-down arguments in favour of the relative/partial distinction is going to be very difficult, however, since most researchers assign almost exactly the same semantic analysis to RAs and partial AAs. For example, the standard view of partial adjectives is that they are existential: a partial AA  $P$  picks out the individuals that satisfy  $P$  to some non-zero degree (Kamp and Rossdeutscher (1994), Yoon (1996), Rotstein and Winter (2004), Kennedy and McNally (2005) etc.) (28-a). However, the corresponding relative adjective analysis of  $P$  would pick out the individuals that satisfy  $P$  to some non-zero *contextually given* degree (perhaps that ‘stands out’ (cf. Kennedy (2007))) (28-b).

- (28) a. **Existential:**  
 $\llbracket \text{wet} \rrbracket = \{x : \exists d : x \text{ is wet to degree } d\}$
- b. **Context-Sensitive:**  
 $\llbracket \text{wet} \rrbracket = \{x : x \text{ is wet to degree } d\}$

It is not clear to me what kind of data (besides examples like (22) above, where a partial adjective’s standard is set to a degree given by context) would be relevant for clearly distinguishing between an analysis in which a partial adjective’s standard is always set to the very first degree on the scale (28-a) and an analysis in which the standard can be set to the first degree, but can also be set to other degrees depending on the context (28-b). Thus, a possible analysis of the difference between the adjectives in (25) and those in (27) is that, while they both take their standard from context, the latter class tends to be able to have their contextual standards set much lower than the former class; the standards associated with the items in (27) can even easily be set right at the  $P/\text{not } P$  border, something that is much more difficult (although not impossible, see Burnett (2011a) and Burnett (2012) for examples) for those in (25). Nevertheless, despite my skepticism about the relative/partial

distinction, in the later part of the paper, I will make some remarks about how to extend my analysis of vagueness and comparatives with relative and total absolute adjectives to partial adjectives, should we later find convincing empirical arguments that they exist as a distinct class.

This being said, the main conclusion that we should draw from the data in this section is that there is an important lexical class distinction between relative adjectives and (at least one class of) absolute adjectives, and we see the effects of this distinction all over the grammar of English<sup>8</sup>.

### 3 Variation in the Presence of Vagueness

As discussed in the introduction, the uncontroversial examples of vague predicates are relative adjectives like *tall*, *long* and *expensive*. We will see in this section that these lexical items allow borderline cases, fuzzy boundaries and, provided certain basic conditions on the domain are met, they give rise to the Sorites. With respect to the discussion of the data, I will take the presence of this cluster of properties as our operational definition of vague language. Thus, we define *vague* with respect to a context as in definition 1.

**Definition 1. Stage-level Vagueness (informal).** A syntactic constituent  $X$  is vague in context  $c$  if  $X$  has borderline cases, fuzzy boundaries and gives rise to the Sorites in  $c$ .

Furthermore, using definition 1, we can define another property that will become important in the latter part of this paper: *potentially vague*.

**Definition 2. Potentially Vague (informal).** A syntactic constituent  $X$  is potentially vague just in case there is some context  $c$  such that  $X$  is vague in  $c$ .

#### 3.1 Vagueness and Relative Adjectives

In this section, I present the empirical phenomenon of vagueness as it is associated with lexical items of the *relative* class. As outlined in any introductory textbook on formal semantics within the Montagovian tradition (ex. Dowty (1979), Heim and Kratzer (1998) etc.), the semantic theories for natural language commonly adopted by linguists consist in (more-or-less straightforward) extensions of the semantic theory of first order logic (FOL). Therefore, I also discuss the ways in which vague predicates appear to be problematic for the types of theories that we generally adopt in the field of natural language semantics.

The first characterization of vague predicates found in the literature, going back to Peirce (1901), if not earlier, is the *borderline cases* property. That is, vague predicates are those that admit borderline cases: objects of which it is unclear whether or not the predicate applies. Consider the following example with the predicate *tall*: If we are in a context where we take the set of American males as the appropriate

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<sup>8</sup>Of course these distinctions are not limited to English. See Burnett (2012) for a discussion of their prevalence in the Indo-European family.

comparison class for *tallness*, we can easily identify the ones that are clearly tall: for example, anyone over 6 feet. Similarly, it is clear that anyone under 5ft9" (the average) is not tall. But suppose that we look at John who is somewhere between 5ft9" and 6ft. Which one of the sentences in (29) is true?

- (29) a. John is **tall**.  
b. John is **not tall**.

For John, a borderline case of *tall*, it seems like the most appropriate answer is either "neither" or "both". In fact, many recent experimental studies on contradictions with borderline cases have found that the "both" and/or "neither" answers seem to be favoured by natural language speakers. For example, [Alxatib and Pelletier \(2010\)](#) find that many participants are inclined to permit what seem like overt contradictions of the form in (30) with borderline cases. Additionally, [Ripley \(2011\)](#) finds similar judgements for the predicate *near*.

- (30) a. Mary is **neither tall nor not tall**.  
b. Mary is **both tall and not tall**.

Clearly, the existence of borderline cases poses a challenge for our classical semantic theories in both logic and linguistics. These systems are all bivalent: there can be no individuals who are both members of a predicate and its negation. Furthermore, these systems all obey the law of excluded middle: there can be no individuals who are members neither of a predicate nor its negation. Thus, we have a puzzle.

A second characterization of vague predicates going back to [Frege \(1904\)](#)'s *Grundgesetze* is the *fuzzy boundaries* property. This is the observation that there are (or appear to be) no sharp boundaries between cases of a vague predicate *P* and its negation. Considering the context described above: If we take a tall person and we start subtracting millimetres from their height, it seems impossible to pinpoint the precise instance where subtracting a millimetre suddenly moves us from the height of a tall person to the height of a not tall person. The fuzzy boundaries property is problematic for our classical semantic theories because we assign set-theoretic structures to predicates and their negations, and these sets have sharp boundaries. In principle, if we line all the individuals in the domain up according to height, we ought to be able to find an adjacent pair in the *tall*-series consisting of a tall person and a not tall person. However, it does not appear that this is possible.

Of course, one way to get around this problem would be to just stipulate where the boundary is, say, at another contextually given value for *tall*; however, if we were to do this, we would be left with the impression that the point at which we decided which of the borderline cases to include and which to exclude was arbitrary. The inability to draw sharp, non-arbitrary boundaries is often taken to be the essence of vagueness (for example, by [Fara \(2000\)](#)), and it is intimately related to another characterization of vague language: vague predicates are those that are *tolerant*. This novel definition of vagueness was first proposed by [Wright \(1975\)](#) as a way to give a more general explanation to the 'fuzzy boundaries' feature; however, versions of this idea have, more recently, been further developed and taken to be at the core of what it means to be a vague expression (ex. [Eklund \(2005\)](#), [Smith \(2008\)](#), [van Rooij](#)

(2010), Cobreros et al. (2010)).

(31) **Definition of Tolerance (Wright 1975)(p. 334):**

“Let  $\Theta$  be a concept related to a predicate,  $F$ , in the following way: that any case to which  $F$  applies may be transformed into a case where it does not apply simply by sufficient change in respect of  $\Theta$ ; colour, for example, is such a concept for ‘red’, size for ‘heap’, degree of maturity for ‘child’, number of hairs for ‘bald’...

- a. Then  $F$  is **tolerant** with respect to  $\Theta$  if there is also some positive degree of change in respect of  $\Theta$  insufficient ever to affect the justice with which  $F$  is applied to a particular case.”

This property is more nuanced than the ‘fuzzy boundaries’ property in that it makes reference to a dimension and to an incremental structure associated with this dimension. The definition in (31) puts an additional constraint on what can be defined as a vague predicate: the distance between the points on the associated dimension must be sufficiently small such that changing from one point to an adjacent one does not affect whether we would apply the predicate. Immediately, we can see that, in the context of American males, *tall* satisfies (31). There is an increment, say 1 mm, such that if someone is tall, then subtracting 1 mm does not suddenly make them not tall. Similarly, adding 1 mm to a person who is not tall will never make them tall.

The observation that relative adjectives are tolerant leads straightforwardly to the observation that these predicates gives rise to a paradox for systems like FOL known as the *Sorites*, or the paradox of the ‘heap’. Formally, the paradox can set up in a number of ways in FOL. A common one found in the literature is (32), where  $\sim_p$  is a ‘little by little’ or ‘indistinguishable difference’ relation.

(32) **The Sorites Paradox**

- a. **Clear Case:**  $P(a_1)$
- b. **Clear Non-Case:**  $\neg P(a_k)$
- c. **Sorites Series:**  $\forall i \in [1, n](a_i \sim_p a_{i+1})$
- d. **Tolerance:**  $\forall x \forall y ((P(x) \wedge x \sim_p y) \rightarrow P(y))$
- e. **Conclusion:**  $P(a_k) \wedge \neg P(a_k)$

Thus, in FOL and other classical systems, as soon as we have a clear case of  $P$ , a clear non-case of  $P$ , and a Sorites series, though *universal instantiation* and repeated applications of *modus ponens*<sup>9</sup> we can conclude that everything is  $P$  and that everything is not  $P$ . We can see that *tall* (for a North American male) gives rise to such an argument. We can find someone who measures 6ft to satisfy (32-a), and we can find someone who measures 5ft6" to satisfy (32-b). In the previous paragraph, we concluded that *tall* is tolerant, so it satisfies (32-d), and, finally, we can easily construct a Sorites series based on height to fulfil (32-c). Therefore, we would expect to be able to conclude that this 5ft6" tall person (a non-borderline case) is both tall

<sup>9</sup>Note that UI is not even necessary for the paradox: we can replace the quantified statements in (32) by individual conditionals and the result is the same; it is the validity of MP that is important for the Sorites.

and not tall. I emphasize that the Sorites is not only a paradox for FOL. As discussed above, the semantic theories that linguists employ all validate bivalence, excluded middle, and modus ponens.

In summary, we identified a context (that of evaluating the height of American males) in which *tall* has borderline cases, fuzzy boundaries, and gives rise to the Sorites paradox. Thus, *tall* is potentially vague in the sense of definition 2. Furthermore, I outlined the aspects of vague language that appear to challenge the kinds of semantic theories that the vast majority of linguists employ. We can therefore conclude that giving an appropriate account of these aspects and their distribution in the grammar is of utmost importance to the proper semantic and pragmatic analysis of adjectives and to the field of natural language semantics, in general.

### 3.1.1 *Relative Adjective Generalization*

It is generally assumed in the literature that relative adjectives are not only potentially vague (vague in some context), but, in fact, universally vague (vague in all contexts). That is, it seems impossible to think of a context in which we can eliminate the borderline cases of *tall* or sharpen its boundaries in a non-artificial way<sup>10</sup>. This observation about relative adjectives can be stated in the terminology presented in this section as the generalization in (33).

(33) **Relative Adjective Generalization:**

If  $P \in RA$ , then for all contexts/situations  $c$ ,  $P$  has borderline cases, fuzzy boundaries, and gives rise to the Sorites in  $c$ .

### 3.2 *Vagueness and Absolute Adjectives*

It has been observed since Ancient Greece that adjectives like *bald* display certain properties that are eerily similar to the properties displayed by *tall* and *long*. For example, if we take a normal case of the use of the word *bald*, talking about men on the street, we can easily identify clear cases of bald men (those with zero hairs on their head) and clear non-cases (those with a full head of hair). However, in this context, *bald* also seems to present the same properties as *tall*. For instance, what about people with a quarter head of hair? Are they bald? Not bald? Both or neither? Thus, in this context, *bald* appears to have borderline cases. Similarly, at what number of hairs does one go from being bald to not bald? The boundaries of *bald* appear fuzzy. Indeed, it seems bizarre to think that there is some point at which adding a single hair to a man's head could take him from being bald to not bald; therefore, *bald* is tolerant in this context. Thus, we have the ingredients for a Sorites-type argument.

As I mentioned, in one tradition, it is taken for granted that, based on their similarities, the puzzling properties of *tall* and *bald* should be given a unified analysis; however, in another, it is argued that they should receive heterogeneous analyses. For example, when considering the 'shifty-ness' of absolute predicates like *bald* or

<sup>10</sup>By 'non-artificial', I mean by means other than saying "By *tall*, I mean "has a height greater than exactly 6ft", and thereby constructing a new precise predicate "taller than exactly 6ft".

examples like those in (34), Kennedy (2007) (p. 24) says, “These examples illustrate a phenomenon that is distinct from vagueness, though typically exists alongside it: imprecision”.

- (34) a. I’m not awake yet.  
 b. The theater is empty tonight  
 (There are only four people in it!)  
 Kennedy (2007) (p. 23)

From what I can see, the most important argument for making this claim is the apparent observation that there exists systematic lexical-class-based variation in the presence of the symptoms of vagueness. In particular, absolute adjectives, unlike relative adjectives, seem to be able to be used in precise manner. I call this observation the *Pinkal-Kennedy* generalization, and it will be the focus of the remaining part of this section.

### 3.3 The Pinkal/Kennedy Generalization

The main challenge for a unified treatment of contextual effects with *tall* and *bald* comes from the observation that absolute adjectives appear to display a different pattern from relative adjectives with respect to the availability of borderline cases, fuzzy boundaries and the Sorites paradox. The difference seems to be that we can sometimes think of contexts or scenarios in which absolute adjectives appear to ‘sharpen up’, i.e. stop being vague. This can be stated as the generalization in (35).

- (35) **Absolute Adjective Generalization:**  
 If  $Q \in AA$ , then there is some context  $c$  such that  $Q$  has no borderline cases, fuzzy boundaries nor gives rise to the Sorites in  $c$ .

To adapt an example from Fara (2000): suppose we are trying to cast a movie biography of the actor Yul Brynner. Brynner is completely bald, and, indeed, his appearance is one of the things he is famous for. Thus, it is very important that the person that we pick to play him be completely bald (have zero hairs on their head). In this context, it would be appropriate to say something like (36).

- (36) The lead actor must be **bald**, but this guy has a hair on his head, so unfortunately, he won’t work.

In this situation, *bald* has no borderline cases, and adding a single hair moves one sharply from *bald* to *not bald*.

As we mentioned above, the behaviour of AAs is different from that of RAs, because RAs seem to be vague in all contexts (cf. (33)). We can state this empirical observation that links lexical class membership to possible lack of vagueness, which I will henceforth call the *Pinkal/Kennedy Generalization*, as in (37).

(37) **The Pinkal/Kennedy Generalization**

Relative adjectives are vague in all contexts; whereas, there exist contexts in which absolute adjectives are not vague.

In [Burnett \(2011a\)](#) and [Burnett \(2012\)](#), I argue that the P/K generalization is empirically incorrect: we can find contexts in which most (if not all) relative adjectives cease being vague; however, nothing in this paper hinges on this point.

The conclusion that Kennedy, in particular, draws from (37) is that this difference justifies having one account for contextual effects with RAs and another (different) account for contextual effects with AAs. However, in this paper, I will assume that the dominant view in the philosophical tradition is correct: we must search for a unified analysis of the puzzling contextual effects associated with *tall* and *bald*. More explicit empirical arguments in favour of this position are given in [Burnett \(2011a\)](#) and [Burnett \(2012\)](#). Therefore, the first puzzle to be solved is the following:

(38) **Contextual Variation in the Presence of Vagueness:**

How can we provide a unified analysis of the puzzling properties of vague language that appear with both relative and absolute adjectives, while, at the same time, account for the contextual variation in the availability of these properties with absolute adjectives?

In section 5, I give a solution to this puzzle.

#### 4 The Existence of Absolute Comparatives

As mentioned in the introduction, one of the focuses of this paper is the scales derived from constituents with an absolute meaning. In particular, we would like to know not only from a technical perspective how it is possible to associate scales with non-context-sensitive constituents, but also why comparatives with members of the absolute class like (39) even exist.

- (39)
- a. Ottawa is **cleaner** than Montreal.
  - b. Closet A is **emptier** than closet B.
  - c. This road is **straighter** than that road.
  - d. Your glass is **more full** than mine is.

The puzzle of the existence of absolute comparatives is the following: as shown in sections 2 and 3, there are many constructions and many contexts in which absolute adjectives have a non-scalar (and precise) meaning. For example, as discussed in [Kyburg and Morreau \(2000\)](#) and [Kennedy \(2007\)](#), inserting an AA in the frame *the - one* forces a non-gradable reading, as does accentuating it ([Unger \(1975\)](#), [Kennedy \(2007\)](#)).

- (40)
- a. My glass is the **empty** one.  
(Only ok if one of the glasses is (close enough to) completely empty.)
  - b. My glass is **EMPTY**, # but it could be emptier.



Note that relative adjectives do not have this property.

- (41) a. My glass is the **tall** one.  
(Ok if both glasses are rather short, but one is taller.)  
b. My glass is **TALL**, but it could be taller.

At the same time, however, as shown in (39), AAs are fine in comparatives, and can combine with a wide range of degree modifiers (42), just like relative adjectives. So what's going on?

- (42) a. John is **very bald**.  
b. Your room is **pretty clean**.

A very common solution to this puzzle in linguistics is to treat the semantics of absolute adjectives as context-sensitive just like relative adjectives (Rotstein and Winter (2004), Kennedy (2007), Toledo and Sassoon (2011), among others). Since, it is proposed that the extension of adjectives like *bald* and *empty* varies across contexts (or, in a degree semantics framework, it is proposed that absolute adjectives are semantically associated with a scale), we can attribute the scale associated with AAs to contextual variation (or argument structure) in exactly the same way as with relative adjectives. Of course, now what remains to be explained is why these constituents sometimes seem to be absolute.

There are a couple of possibilities available, but, by far, Kennedy (2007)'s solution has been the most influential. He proposes that what enforces an absolute meaning with most uses of AAs is a meta-grammatical principle called *Interpretative Economy* (43). In Kennedy's framework, AAs are associated with scales with endpoints (i.e. the endpoint of the *empty* scale is zero objects), thus, (43) is designed to force the interpretation of an AA to be as close to the endpoint of the scale as possible.

- (43) **Interpretative Economy:**  
Maximize the contribution of the conventional meanings of the elements of a sentence to the computation of its truth conditions.  
(Kennedy 2007) (p. 36)

But, this seems unsatisfactory. First of all, as Kennedy himself notes (ftn.31), the status of this principle in the grammar or its place in a more general theory of cognition is unclear. Indeed, it is also unclear why the aspect of the 'conventional meaning' of an absolute adjective that is maximized must be the endpoint of its scale. Why do we not maximize the fact that AAs are context-sensitive, which, surely, is also part of their conventional meaning? Furthermore, treating AAs like relative adjectives to whom we can assign no meaning in the absence of a comparison class seems to miss an important generalization about the intuitions that speakers have about the meaning of words of their language. To quote Récanati (2010) (p. 117), who sums up the puzzle nicely with respect to the context-sensitivity of *empty*,

As a matter of fact, we know perfectly well which property the adjective *empty* expresses. It is the property (for a container) of not containing anything, of being devoid of contents. That is how we define *empty*. Note

that this is an absolute property, a property which a container has or does not have. Either it contains something, or it does not contain anything. So the property which the adjective expresses and which determines its extension is not a property that admits of degrees. How, then, can we explain the gradability of the adjective ?

In what follows, I will provide a solution to this puzzle that crucial makes use of the empirical fact that absolute adjectives are potentially vague.

## 5 A Unified Analysis of Vague Scalar Adjectives

In this section, I propose a unified analysis of the vagueness associated with both absolute and relative adjectives. Based in the data discussed above, such an analysis ought have the following features:

1. **Unified Analysis:** Borderline cases etc. arise through the same mechanisms with both relative and absolute adjectives.
2. **Class Distinctions:** At some level, there must be a difference between RAs and AAs such that it is possible, in principle, to account for the non-vagueness-related data in section 2.
3. **Possible Precision:** It should be possible for absolute adjectives to be assigned a precise meaning in some context.

My main proposal is the following:

1. **Scalar adjectives have classical (precise) semantics.** Adjectives of both classes are assigned semantic denotations that can be modelled in straightforward extensions of first order logic.
2. These constituents are assigned secondary values which are obtained through context-dependent computation that takes their classical semantic denotations as input.
3. **Scalar adjectives have non-classical pragmatics.** These secondary pragmatic values create the puzzling properties of vague language.

The section is laid out as follows: as mentioned, I will adopt the *Tolerant, Classical, Strict* account of vague language, and thus, my analysis will consist of enriching the TCS framework to account for the differences between *tall* and *bald*. I therefore first give a brief introduction to the TCS system and present the notation that I will be adopting in the rest of the paper. I then outline the framework that I will be using for the semantic analysis of relative and absolute adjectives: Delineation semantics. Finally, I give a unified analysis of vagueness in the adjectival domain.

Before I give the semantic/pragmatic analysis, however, some syntactic remarks are in order. The analyses that I will give in this article will deal with a very small range of syntactic constructions: copular sentences with the positive form of relative adjectives (44-a), the positive form of absolute adjectives (44-b), comparatives with

relative adjectives (44-c), comparatives with absolute adjectives (44-d), and, finally, positive non-scalar adjectives (44-e).

- (44) a. John is tall.  
 b. John is bald.  
 c. John is taller than Peter.  
 d. John is balder than Peter.  
 e. John is dead.

Importantly, we will only deal with singular individual denoting subjects like *John* or *Mary*. Vagueness with plural and quantificational subjects is examined within the TCS framework in Burnett (2011b) and Burnett (2012).

Thus, syntactically, we are interested in three basic types of lexical items:

1. Singular individual denoting subjects (like *John*), which will often be notated by lowercase letters of the alphabet ( $a, b, c, j, m \dots$ ).
2. Predicates of the relative adjective class (RA), which will generally be notated by members of the  $P$  series:  $P, P_1, P_2 \dots$
3. Predicates of the absolute adjectives class (AA), which will generally be notated by members of the  $Q$  series:  $Q, Q_1, Q_2 \dots$

I will also refer to the entire class of scalar adjectives as  $SA$  ( $RA \cup AA = SA$ ). Often, if the relative/absolute distinction is irrelevant for a particular definition, I will use members of the  $P$  series to notate members of  $SA$ . I do not believe this will cause confusion.

### 5.1 *Tolerant, Classical, Strict*

In this section, I outline Cobreros et al. (2010)'s *Tolerant, Classical, Strict* framework. This system was developed as a way to preserve the intuition that vague predicates are *tolerant* (i.e. satisfy  $\forall x \forall y [P(x) \ \& \ x \sim_p y \rightarrow P(y)]$ , where  $\sim_p$  is an indifference relation for a predicate  $P$ ), without running into the Sorites paradox. Cobreros et al. (2010) adopt a non-classical logical framework with three notions of satisfaction: classical truth, tolerant truth, and its dual, strict truth. Formulas are tolerantly/strictly satisfied based on classical truth and predicate-relative, possibly non-transitive *indifference relations*. For a given predicate  $P$ , an indifference relation,  $\sim_p$ , relates those individuals that are viewed as sufficiently similar with respect to  $P$ . For example, for the predicate *tall*,  $\sim_{tall}$  would be something like the relation "not looking to have distinct heights". In this framework, we say that *John is tall* is tolerantly true just in case John has a very similar height to someone who is classically tall (i.e. has a height greater than or equal to the contextually given 'tallness' threshold).

An important point to note about TCS, in its formulation in the Cobreros et al. (2010) paper, is that it makes no distinctions between lexical classes of scalar adjectives<sup>11</sup>. Thus, one of the main features of my proposal below is to introduce

<sup>11</sup>Or between scalar and non-scalar adjectives, or, for that matter, between adjectival predicates and

scale-structure distinctions into this framework. The aspects of the system that are relevant for the present analysis are defined (using the notation adopted in this paper) as follows: For the semantics, we define three notions of truth. The first one corresponds to truth in classical FOL (*c(lassical)-truth*), and, in this paper, I will assume that classical denotations correspond to the semantic denotations of linguistic expressions. The two notions of truth that are novel: *t-truth* and its dual *s-truth*, are pragmatic notions.

We first define our c(lassical) models.

**Definition 3. C(lassical) Model.** A c-model is a tuple  $\langle D, \llbracket \cdot \rrbracket^c \rangle$  where  $D$  is a non-empty domain of individuals and  $\llbracket \cdot \rrbracket^c$  is an interpretation function for the non-logical vocabulary.

1. For an individual denoting subject DP  $a$ ,  $\llbracket a \rrbracket^c \in D$ .
2. For  $P \in SA$ ,  $\llbracket P \rrbracket^c \in \mathcal{P}(D)$ .

We then extend our classical models to tolerant models by adding the  $\sim$  function.

**Definition 4. T(olerant) Model.** A t-model is a tuple  $\langle D, \llbracket \cdot \rrbracket^c, \sim \rangle$ , where  $\langle D, \llbracket \cdot \rrbracket^c \rangle$  is a c-model and  $\sim$  is a function that takes any predicate  $P \in SA$  to a binary relation  $\sim_P$  on  $D$ . For any  $P$ ,  $\sim_P$  is reflexive and symmetric (but possibly not transitive)<sup>12</sup>.

*C-truth* is defined as classical truth in either a c-model or a t-model.

**Definition 5. c-truth in a model.** Let  $M$  be either a c-model such that  $M = \langle D, \llbracket \cdot \rrbracket^c \rangle$  or a t-model such that  $M = \langle D, \llbracket \cdot \rrbracket^c, \sim \rangle$ .

- (45) For a subject  $a$  and  $P \in SA$ ,  
 $\llbracket a \text{ is } P \rrbracket^c = 1$  iff  $\llbracket a \rrbracket^c \in \llbracket P \rrbracket^c$ .

Tolerant and strict denotations of constituents are defined based on classical denotations and contextually given indifference relations.

**Definition 6. Tolerant denotation.** ( $\llbracket \cdot \rrbracket^t$ )

1. For a subject  $a$ ,  $\llbracket a \rrbracket^t = \llbracket a \rrbracket^c$ .
2. For  $P \in SA$ ,  $\llbracket P \rrbracket^t = \{x : \exists d \sim_P x : d \in \llbracket P \rrbracket^c\}$ .

**Definition 7. Strict denotation.** ( $\llbracket \cdot \rrbracket^s$ )

1. For a subject  $a$ ,  $\llbracket a \rrbracket^s = \llbracket a \rrbracket^c$ .
2. For  $P \in SA$ ,  $\llbracket P \rrbracket^s = \{x : \forall d \sim_P x, d \in \llbracket P \rrbracket^c\}$ .

intransitive verbs. This is because the 2010 paper provides a non-classical semantics for a very slightly extended version of the language of first order predicate logic, which does not make these distinctions.

<sup>12</sup>In Cobreros et al. (2010), it is assumed that all instances of  $\sim_P$  are symmetric, but see Burnett (2012) for arguments that this does not always seem to be case.

A first point to note is that, to be consistent with [Cobrerros et al. \(2010\)](#) (and also for simplicity), subject DPs are interpreted ‘crispily’: their tolerant and strict denotations are identical to their classical denotations. This, of course, is not logically necessary, and, in fact, a TCS-based analysis of vague DP subjects is given in [Burnett \(2011b\)](#).

Like classical truth, tolerant and strict truth are defined simply as membership in the tolerant/strict denotation.

**Definition 8.** t-truth and s-truth. Let  $M$  be a t-model.

1.  $\llbracket a \text{ is } P \rrbracket^t = 1$  iff  $\llbracket a \rrbracket^t \in \llbracket P \rrbracket^t$ .
2.  $\llbracket a \text{ is } P \rrbracket^s = 1$  iff  $\llbracket a \rrbracket^s \in \llbracket P \rrbracket^s$ .

Finally, negation is defined as in [definition 9](#): classical negation is defined as complement; however, tolerant and strict negated predicates are inter-defined as below.

**Definition 9. Negation.** For a predicate  $P$ ,

1.  $\llbracket \text{not } P \rrbracket^c = \{x : x \notin \llbracket P \rrbracket^c\}$ .
2.  $\llbracket \text{not } P \rrbracket^t = \{x : x \notin \llbracket P \rrbracket^s\}$ .
3.  $\llbracket \text{not } P \rrbracket^s = \{x : x \notin \llbracket P \rrbracket^t\}$ .

TCS explains the puzzling properties of vague language in the following way. Firstly, although classical negation partitions the domain (like it does in FOL), tolerant negation actually allows for individuals to be members of **both**  $\llbracket P \rrbracket^t$  and  $\llbracket \text{not } P \rrbracket^t$ . These are the borderline cases. The reason that we have difficulty deciding whether a borderline individual is part of a predicate’s extension or anti-extension is that such an individual is actually part of both sets. In other words, at the level of tolerant truth, TCS is paraconsistent: contradictions involving borderline cases do not result in explosion (like they do in classical logic). Secondly, TCS preserves the intuition behind the fuzzy boundaries/tolerance property because the principle of tolerance ((32-d)) is, in fact, valid at the level of tolerant truth. Note that it is neither classically valid nor strictly valid. Finally, using the consequence relation favoured by [Cobrerros et al. \(2010\)](#)<sup>13</sup>,  $\models^{st}$ , i.e. reasoning from strict premises to tolerant conclusions, the Sorites argument is valid. However, since the principle of tolerance is not strictly valid, the Sorites argument is unsound and therefore does not result in paradox.

With TCS as our approach to vague language, I turn to theories of the semantics of scalar constituents. In the next section, I will present one approach to the (classical) semantics of scalar adjectives and then show how we can integrate its insights into TCS.

<sup>13</sup>The TCS actually gives rise to 9 possible consequence relations:  $\models^{tt}, \models^{cc}, \models^{ss}, \models^{st}, \models^{ct}$  etc. [Cobrerros et al. \(2010\)](#) propose that the one that is most relevant for the analysis of vagueness in natural language is  $\models^{st}$  because the deduction theorem and modus ponens are both valid arguments in this system.

## 5.2 Delineation Semantics

Delineation semantics is a framework for analyzing the semantics of gradable expressions that takes the observation that they are context sensitive to be their key feature. A delineation approach to the semantics of positive and comparative constructions was first proposed by Klein (1980), and has been further developed by van Benthem (1982), Keenan and Faltz (1985), Larson (1988), Klein (1991), van Rooij (2011b), Doetjes (2010), van Rooij (2011a), and Doetjes, Constantinescu, and Soucková (2011), among others. In what follows, I will present a very basic version of the theory because it will be sufficient to account for the data discussed in this paper. However, presumably, more enriched theories, such as those proposed by the authors cited above, will be necessary to account for the wide range of scale-based constructions in natural language.

In this framework, scalar adjectives denote sets of individuals and, furthermore, they are evaluated with respect to comparison classes<sup>14</sup>, i.e. subsets of the domain. The basic idea is that the extension of a gradable predicate can change depending on the set of individuals that it is being compared with. For example, consider the predicate *tall* and Klein (1980) (p. 18)'s graphic in figure 1.



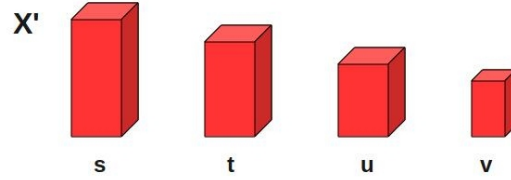
Figure 1: Two-element (minimal) comparison class  $X$

Suppose the rightmost bar is called  $v$ . If we apply the predicate *tall* to the elements in the minimal two-element comparison class  $X = \{u, v\}$ , then the classical extension of *tall* in  $X$ , written  $\llbracket \text{tall} \rrbracket_X^c$ , would be  $\{u\}$ , and  $\llbracket \text{not tall} \rrbracket_X^c = \{v\}$ . Now consider the larger comparison class  $X'$  (also from Klein (1980), p.18).

If we apply the predicate *tall* in  $X'$ , despite the fact that their actual sizes have not changed, it is conceivable that both  $u$  and  $v$  could now be in  $\llbracket \text{not tall} \rrbracket_{X'}^c$ . As we will see below, the classical semantics of the comparative will be defined based on looking at how scalar predicates apply to individuals across comparison classes.

<sup>14</sup>Comparison classes are important in both delineation semantics and degree semantics. In fact, they are generally viewed unavoidable in any theory of scalar adjectives (Klein 1980). Note however that the comparison classes used in modern degree and delineation semantics are purely given by context, not contributed by lexical material like the *for* phrase in (i). See Fara (2000) and Kennedy (2007) for discussion.

(i) John is tall **for a basketball player**.

Figure 2: Four-element comparison class  $X'$ 

However, first, we define our comparison class c-models and c-truth in them.

**Definition 10. Comparison class c-model.** A CC c-model is a tuple  $\langle D, CC, \llbracket \cdot \rrbracket^c \rangle$  where  $D$  is a non-empty domain of individuals and  $CC$  is the set of comparison classes such that  $CC = \mathcal{P}(D)$ . Furthermore,  $\llbracket \cdot \rrbracket^c$  is an interpretation function for the non-logical vocabulary.

1. For an individual denoting subject DP  $a$ ,  $\llbracket a \rrbracket^c \in D$ .
2. For  $P \in SA$ , for every  $X \in CC$ ,  $\llbracket P \rrbracket_X^c \subseteq X$ .

As mentioned, (c-)truth in a CC model is always given with respect to a comparison class. For a given utterance of a sentence containing the positive form of a scalar adjective, the relevant comparison class against which the statement is evaluated is given purely by context.

**Definition 11. Classical semantics of the positive form.** For a subject DP  $a$ ,  $P \in SA$ , and a contextually given  $X \in CC$ ,

$$(46) \quad \llbracket a \text{ is } P \rrbracket_X^c = 1 \text{ iff } \llbracket a \rrbracket^c \in \llbracket P \rrbracket_X^c.$$

Given that for individual denoting subjects,  $\llbracket a \rrbracket^c = \llbracket a \rrbracket^t = \llbracket a \rrbracket^s$ , to keep things simple, I'll omit the  $\llbracket \cdot \rrbracket^{s/c/t}$  brackets around subjects in the definitions that follow.

Unlike degree semantics, delineation semantics takes the positive form (defined in definition 11) as basic and derives the semantics of the comparative form from quantification over CCs. Informally, *John is taller than Mary* is true just in case there is some comparison class with respect to which John counts as tall and Mary counts as not tall. Thus, in the example in figures 1 and 2,  $u$  is taller than  $v$  since  $u$  is tall in  $X$  and  $v$  is not tall in  $X$ . Supposing both  $u$  and  $v$  are in the anti-extension of *tall* in  $X'$ , the comparison class in figure 2 is not helpful for deciding whether  $\langle u, v \rangle$  is a member of the *taller than* relation.

**Definition 12. Classical semantics for the comparative.** For two DPs  $a, b$  and  $P \in SA$ ,  $\llbracket a \text{ is } P\text{-er than } b \rrbracket^c = 1$  iff  $a >_P^c b$ , where  $>_P^c$  is defined as:

$$(47) \quad x >_P^c y \text{ iff there is some } X \in CC \text{ such that } x \in \llbracket P \rrbracket_X^c \text{ and } y \notin \llbracket P \rrbracket_X^c.$$

As it stands, the analysis of the comparative in definition 12 allows some very strange and un-comparative-like relations, if we do not say anything about how the extensions of gradable predicates can change in different comparison classes. For example, suppose in the CC  $\{a, b\}$ ,  $a \in \llbracket P \rrbracket_{\{a,b\}}^c$  and  $b \notin \llbracket P \rrbracket_{\{a,b\}}^c$ . So  $a >_p^c b$ . And suppose moreover that, in the larger CC  $\{a, b, c\}$ ,  $b \in \llbracket P \rrbracket_{\{a,b,c\}}^c$  and  $a \notin \llbracket P \rrbracket_{\{a,b,c\}}^c$ . So  $b >_p^c a$ . But clearly, natural language comparatives do not work like this: If John is {taller, fatter, balder...} than Mary, Mary cannot also be {taller, fatter, balder...} than John. In other words,  $>_p^c$  must be asymmetric.

A solution to this problem involves imposing some constraints on how predicates like *tall* can be applied in different CCs. Klein proposes two conditions to ensure that the kind of situation just described cannot occur. In this work, however, I will adopt another set of constraints on the application of gradable predicates, those presented in van Benthem (1982) and van Benthem (1990). Van Benthem proposes three axioms governing the behaviour of individuals across comparison classes. They are the following (presented in my notation):

For  $x, y \in D$  and  $X \in CC$  such that  $x \in \llbracket P \rrbracket_X^c$  and  $y \notin \llbracket P \rrbracket_X^c$ ,

- (48) **No Reversal (NR):** There is no  $X' \in CC$  such that  $y \in \llbracket P \rrbracket_{X'}^c$  and  $x \notin \llbracket P \rrbracket_{X'}^c$ .  
(49) **Upward difference (UD):** For all  $X'$ , if  $X \subseteq X'$ , then there is some  $z, z' : z \in \llbracket P \rrbracket_{X'}^c$  and  $z' \notin \llbracket P \rrbracket_{X'}^c$ .  
(50) **Downward difference (DD):** For all  $X'$ , if  $X' \subseteq X$  and  $x, y \in X'$ , then there is some  $z, z' : z \in \llbracket P \rrbracket_{X'}^c$  and  $z' \notin \llbracket P \rrbracket_{X'}^c$ .

**No Reversal** states that if  $x >_p^c y$ , there is no  $X'$  such that  $y$  is in  $\llbracket P \rrbracket_{X'}^c$ , but  $x$  is not. **Upward Difference** states that if, in the comparison class  $X$ , there is a *P/not P* contrast, then a *P/not P* contrast is preserved in every larger CC. Finally, **Downward Difference** says that if in some comparison class  $X$ , there is a *P/not P* contrast involving  $x$  and  $y$ , then there remains a contrast in every smaller CC that contains both  $x$  and  $y$ . An important consequence of DD is that  $>_p^c$  has the following property: if  $x >_p^c y$ , then, if we look at the minimal two-element comparison class (i.e. if we compare  $x$  directly with  $y$ ),  $x$  will be *P* and  $y$  will be *not P*.

**Theorem 13. Two-element reducibility.** *If  $x >_p^c y$ , then  $x \in \llbracket P \rrbracket_{\{x,y\}}^c$  and  $y \notin \llbracket P \rrbracket_{\{x,y\}}^c$ .*

*Proof.* Suppose  $x >_p^c y$  to show  $x \in \llbracket P \rrbracket_{\{x,y\}}^c$  and  $y \notin \llbracket P \rrbracket_{\{x,y\}}^c$ . Since  $x >_p^c y$ , there is some  $X \in CC$  such that  $x \in \llbracket P \rrbracket_X^c$  and  $y \notin \llbracket P \rrbracket_X^c$ . Clearly  $\{x, y\} \subseteq X$ . So, by Downward Difference, there is some  $z, z' \in \{x, y\}$  such that  $z \in \llbracket P \rrbracket_{\{x,y\}}^c$  and  $z' \notin \llbracket P \rrbracket_{\{x,y\}}^c$ . By No Reversal,  $x \in \llbracket P \rrbracket_{\{x,y\}}^c$  and  $y \notin \llbracket P \rrbracket_{\{x,y\}}^c$ .  $\square$

Finally, van Benthem shows that these axioms give rise to *strict weak orders*: irreflexive, transitive and almost connected relations.

**Definition 14. Strict weak order.** A relation  $>$  is a strict weak order just in case  $>$  is **irreflexive, transitive, and almost connected**.

The definitions of *irreflexivity*, *transitivity* and *almost connectedness* are given below.



**Definition 15. Irreflexivity.** A relation  $>$  is irreflexive iff there is no  $x \in D$  such that  $x > x$ .

**Definition 16. Transitivity.** A relation  $>$  is transitive iff for all  $x, y, z \in D$ , if  $x > y$  and  $y > z$ , then  $x > z$ .

**Definition 17. Almost Connectedness.** A relation  $>$  is almost connected iff for all  $x, y \in D$ , if  $x > y$ , then for all  $z \in D$ , either  $x > z$  or  $z > y$ .

As discussed in Klein (1980), van Benthem (1990) and van Rooij (2011b), strict weak orders (also known as *ordinal scales* in measurement theory) intuitively correspond to the types of relations expressed by many kinds of comparative constructions<sup>15</sup>. For example, one cannot be taller than oneself; therefore  $>_{tall}^c$  should be irreflexive. Also, if John is taller than Mary, and Mary is taller than Peter, then we know that John is also taller than Peter. So  $>_{tall}^c$  should be transitive. Finally, suppose John is taller than Mary. Now consider Peter. Either Peter is taller than Mary (same height as John or taller) or he is shorter than John (same height as Mary or shorter). Therefore,  $>_{tall}^c$  should be almost connected. Thus, theorem 18 is an important result in the semantic analysis of comparatives, and it shows that scales associated with gradable predicates can be constructed from the context-sensitivity of the positive form and certain axioms governing the application of the predicate across different contexts.

**Theorem 18. Strict Weak Order.** For all  $P \in SA$ ,  $>_P^c$  is a strict weak order.

*Proof.* van Benthem (1982); van Benthem (1990), p. 116. □

In summary, the delineation approach to the semantics of scalar predicates makes use of no mechanisms other than individuals and comparison classes (sets of individuals). The semantic denotation of the positive form of the adjective is context-sensitive and varies between CCs. The semantic denotation of the comparative is not context sensitive and is defined based on existential quantification over comparison classes. Finally, with van Benthem's axioms constraining the assignment of predicates across comparison classes, we can derive the strict weak orders that are the scales lexicalized by the comparative.

### 5.3 Solution to Puzzle 1

This section gives a unified analysis of the vagueness of *tall* and *bald* within a delineation approach to the classical semantics of scalar predicates.

#### 5.3.1 Semantics

We adopt comparison class c-models (51-a) and the semantic analysis proposed in the Klein-ian framework for the semantic (classical) interpretation of the positive forms of both classes of adjectives.

<sup>15</sup>Note that we're talking only about explicit comparatives like *John is taller than Mary*. See van Rooij (2011a) for arguments that implicit comparatives, like *John is tall compared to Mary* lexicalize semi-orders. Furthermore, see van Rooij (2011b) and Sassoon (2010) for arguments that more restrictive orders are necessary to account for the interpretation of measure phrases in comparatives.

(51) **C-Semantics**

- a. A CC c-model is a tuple  $\langle D, CC, \llbracket \cdot \rrbracket^c \rangle$  where  $D$  is a non-empty domain of individuals and  $CC$  is the set of comparison classes such that  $CC = \mathcal{P}(D)$ . Furthermore,  $\llbracket \cdot \rrbracket^c$  is an interpretation function for the non-logical vocabulary.
1. For an individual denoting subject DP  $a$ ,  $\llbracket a \rrbracket^c \in D$ .
  2. For  $P \in SA$ , for every  $X \in CC$ ,  $\llbracket P \rrbracket_X^c \subseteq X$ .
- b. For a subject DP  $a$ ,  $P \in SA$ , and a contextually given  $X \in CC$ ,  $\llbracket a \text{ is } P \rrbracket_X^c = 1$  iff  $a \in \llbracket P \rrbracket_X^c$ .

We saw in section 2 that the absolute/relative distinction is an important syntactic and semantic distinction, and, thus, we want some way to capture it. I proposed furthermore in section 3 that we want a single explanation for the appearance of the puzzling properties associated with vague language with both classes of adjectives. As a way to fulfil these two analytical goals, I propose that the difference that gives rise to the patterns discussed in section 2 lies at the level of the semantics of the positive forms of these expressions, not at the level of their pragmatics. In the TCS framework, then, this can be modelled by proposing that positive and absolute adjectives differ at the classical semantic level.

However, so far, nothing distinguishes the classical semantics of relative adjectives from that of absolute adjectives. An idea that has been present in the literature for a long time, and has recently been incarnated in, for example, [Kennedy and McNally \(2005\)](#) and [Récanati \(2010\)](#), is that unlike *tall* or *long* that have a context sensitive meaning, adjectives like *bald*, *empty* or *straight* are not context sensitive (hence the term *absolute* adjective). That is, in order to know who the bald people are or which rooms are empty, we don't need compare them to a certain group of other individuals, we just need to look at their properties. To incorporate this idea into the delineation approach, I propose (following a suggestion from [van Rooij \(2011c\)](#)) that, in a semantic framework based on comparison classes, what it means to be non-context-sensitive is to have your denotation be invariant across classes. Thus, for an absolute adjective  $Q$  and a comparison class  $X$ , it suffices to look at what the extension of  $Q$  is in the maximal CC, the domain  $D$ , in order to know what  $\llbracket Q \rrbracket_X^c$  is. I therefore propose that an additional axiom governs the classical interpretation of the members of the absolute class that does not apply to the relative class: the *absolute adjective axiom (AAA)*.

- (52) **Absolute Adjective Axiom.** If  $Q \in AA$ , then for all  $X \in CC$  and  $x \in X$ ,  $x \in \llbracket Q \rrbracket_X^c$  iff  $x \in \llbracket Q \rrbracket_D^c$ .

In other words, the semantic denotation of an absolute adjective is set with respect to the total domain, and then, by the AAA, the interpretation of  $Q$  in  $D$  is replicated in each smaller comparison class.

In summary, I have made the following proposals about the semantics of scalar adjectives:

1. The semantic denotation of relative adjectives varies depending on compari-

son class.

- It is constrained only by axioms that generate strict weak orders, such as those proposed by Klein (1980) or van Benthem (1982).
2. The semantic denotation of absolute adjectives is constant across comparison classes.
- It is constrained by the AAA.

The AAA is very powerful. In fact, even without any of van Benthem’s axioms, we can prove that the scales associated with AAs are strict weak orders.

**Theorem 19.** *For all  $Q \in AA$ ,  $>_Q^c$  is a strict weak order.*

*Proof. Irreflexivity.*  $x$  cannot be both in  $\llbracket Q \rrbracket_X^c$  and not in  $\llbracket Q \rrbracket_X^c$ . *Transitivity.* Trivially. *Almost Connected.* Let  $x, y, z \in D$  and suppose  $x >_Q^c y$ . Since, by the AAA, all classical denotations are subsets of  $\llbracket Q \rrbracket_D^c$ , we have two cases: 1) if  $z \in \llbracket Q \rrbracket_Q^c$ , then  $z >_Q^c y$ , and 2) if  $z \notin \llbracket Q \rrbracket_Q^c$ , then  $x >_Q^c z$ .  $\square$

Of course, as shown in the proof of theorem 19, although they are technically strict weak orders, the scales that the semantic denotations of absolute constituents give rise to are very small, essentially trivial. As shown by theorem 20, the scales associated with absolute adjectives under my analysis are actually homomorphic to the two-element boolean algebra (sometimes written 2).

**Theorem 20.** *If  $Q \in AA$ ,  $>_Q^c$  is homomorphic to the two element boolean algebra.*

*Proof.* Consider the function  $h : D \rightarrow 2$ .

$$(53) \quad \text{For all } x \in D, h(x) = 1 \text{ iff } x \in \llbracket Q \rrbracket_D^c.$$

Show for all  $\langle x, y \rangle \in <_Q^c$ , if  $x >_Q^c y$  then  $h(x) >_2 h(y)$ . Immediately from the definition of  $h$ .  $\square$

As we will see in later sections, this fact will have implications for the analysis of comparatives formed with absolute adjectives; however, for the moment we can just note it, and summarize the proposed axiom sets governing the semantics of scalar adjectives in table 2. Since the principal effect of van Benthem’s three axioms is to generate strict weak orders, and since there are other axioms sets that do this, I will group them together as the set SWO in the table below<sup>16</sup>.

Axiom	RA	AA
SWO	✓	(✓)
AAA	×	✓

Table 2: Semantic axioms for scalar adjectives

Thus, we have an important difference between the absolute class and the relative class at the semantic level.

<sup>16</sup>Note that the SWO axioms with AAs can be proved as theorems, given theorem 19 and van Benthem (1990)’s theorem 1.5.4, (p.117).

### 5.3.2 Pragmatics

In Klein (1980), vagueness is a semantic phenomenon; that is, Klein adopts a supervaluationist account and proposes that the semantic denotation of a scalar term can contain gaps within a comparison class. However, as discussed in Pinkal (1995) and Burnett (2011a), at least in the way adopted by authors such as Klein, supervaluationism does not account for the Pinkal/Kennedy generalization. Furthermore, in the literature, there are many arguments in favour of being skeptical of super- and subvaluationist approaches in general<sup>17</sup>. Therefore, despite adopting Klein's semantic analysis of relative adjectives, I reject both his (non-distinct) analysis of absolute adjectives and his approach to adjectival vagueness.

Instead, the basic idea is to build pragmatic structures on top of semantic structures in the way done in Cobreros et al. (2010), and use the properties of this logic to model the properties of vague language. Since, as proposed in the previous section, the underlying semantic structures associated with absolute and relative adjectives are different, there will be some effects of this difference on the availability of vagueness with the respective adjective classes, even if the processes that create the appearance of fuzziness are the same in both cases.

We therefore extend our CC c-models to CC t-models by adding the function  $\sim$  in the way shown in definition 21.

**Definition 21. Comparison Class t-model.**  $\langle D, CC, \llbracket \cdot \rrbracket^c, \sim \rangle$ , where  $\langle D, CC, \llbracket \cdot \rrbracket^c \rangle$  is a CC c-model and  $\sim$  is a two-place function that maps an ordered pair  $\langle X, P \rangle$  to a binary relation  $\sim_P^X$  that is reflexive, symmetric (and possibly not transitive).

Note that now, instead of mapping a predicate to an indifference relation,  $\sim$  maps a predicate and a comparison class to an indifference relation on the members of the class. Thus, tolerant and strict denotations are now relativized to comparison classes, and, as such, they are defined as in definition 22.

**Definition 22. Tolerant/Strict CC denotations.** For  $P \in SA$  and  $X \in CC$ ,

1.  $\llbracket P \rrbracket_X^t = \{x : \exists d \sim_P^X x : d \in \llbracket P \rrbracket_X^c\}$ .
2.  $\llbracket P \rrbracket_X^s = \{x : \forall d \sim_P^X x, d \in \llbracket P \rrbracket_X^c\}$ .

Finally, the tolerant/strict semantics for the positive form of an adjective with respect to a comparison class can be given as in 23.

**Definition 23. Positive form.** For a contextually given  $X \in CC$ ,

1.  $\llbracket a \text{ is } P \rrbracket_X^t = 1$  iff  $a \in \llbracket P \rrbracket_X^t$ .
2.  $\llbracket a \text{ is } P \rrbracket_X^s = 1$  iff  $a \in \llbracket P \rrbracket_X^s$ .

The analysis in definition 23 is simply a straightforward implementation of the Klein-ian approach to the semantics of scalar adjectives within a TCS account of the pragmatics of vague predicates. Thus, in a particular situation with a particular

<sup>17</sup> See Fara (2000) (and subsequent work), Smith (2008) and Hyde (2010) for critiques of the s'valuationist approach to vagueness.

comparison class  $X$ , a predicate  $P$  of either semantic class can have borderline cases (objects that in both  $\llbracket P \rrbracket_X^t$  and  $\llbracket \text{not } P \rrbracket_X^t$ ), and, provided the members of  $X$  are related by  $\sim_P^X$  in a way that forms a Soritical series, we can construct a Sorites paradox. Thus, in this analysis, both absolute adjectives and relative adjectives can be vague.

Within this framework, we can also arrive at a more formal statement of the *potentially vague* property that was briefly presented above (definition 2), repeated in definition 24.

**Definition 24. Potentially vague (informal).** A syntactic constituent  $X$  is potentially vague just in case there is some context  $c$  such that  $X$  has borderline cases, fuzzy boundaries, and gives rise to the Sorites in  $c$ .

We can unpack this statement within Klein-ian TCS (as it applies to constituents of the adjectival class) as the conditions in definition 25.

**Definition 25. Potentially vague adjective (formal)** An adjective  $P$  is potentially vague just in case there is some CC t-model  $M$  such that there is some  $X \in CC$  such that,

1. **Clear Case:** There is some  $a_1 \in X$  such that  $a_1 \in \llbracket P \rrbracket_X^s$ .
2. **Clear Non-Case:** There is some  $a_n \in \llbracket \text{not } P \rrbracket_X^s$ .
3. **Sorites Series:** There are  $a_1 \dots a_n \in X$  such that  $a_1 \sim_P^X a_2$ , and  $a_2 \sim_P^X a_3 \dots a_{n-2} \sim_P^X a_{n-1}$ , and  $a_{n-1} \sim_P^X a_n$ .

In the case where the conditions in definition 25 are met for an adjectival predicate (or, indeed, as I argue in Burnett (2012), for a predicate of any syntactic category), by the definition of  $\llbracket \llbracket \rrbracket^t$ , the tolerant extension of a predicate  $P$  will be strictly larger than its classical extension and so the extensions of  $P$  and  $\text{not } P$  will overlap, creating borderline cases. Furthermore, because elements that are classically  $P$  are related to classically  $\text{not } P$  elements by  $\sim_P^X$ ,  $\llbracket P \rrbracket_X^t$  bleeds into  $\llbracket \text{not } P \rrbracket_X^c$ , and thus, the boundaries of  $P$  appear fuzzy.

Using the semantic and pragmatic analysis for relative and absolute adjectives that I just presented and definition 25, we can prove that both classes of scalar adjectives are potentially vague.

**Theorem 26. Vagueness of relative adjectives.** For  $P \in RA$ ,  $P$  is potentially vague.

*Proof.* Consider the CC t-model  $M$  such that  $D = \{a, b, c, d\}$ . Consider  $X \in CC$  such that  $X = \{a, b, c, d\}$ . Suppose  $\llbracket P \rrbracket_X^c = \{a, b\}$ . Suppose  $\sim_P^X = \{\langle a, b \rangle, \langle b, c \rangle, \langle c, d \rangle\} +$  reflexivity and symmetry. Suppose furthermore that  $a >_P^c b >_P^c c >_P^c d$  in  $M$ . Therefore,

1. **Clear Case:**  $a \in \llbracket P \rrbracket_X^s$ .
2. **Clear Non-Case:**  $d \in \llbracket \text{not } P \rrbracket_X^s$ .
3. **Sorites Series:** The sequence  $\langle a, b, c, d \rangle$ .

Therefore,  $P$  is potentially vague. □

The argument applies in almost an exactly parallel manner to absolute adjectives. The only difference is that, since as shown in theorem 20, the  $>_Q^c$  relations for absolute adjectives are trivial, we do not yet have a way to order individuals in a comparison class such that the ones satisfying the ‘Sorites series’ condition actually form a *sequence* rather than just an unordered set. The second part of the paper will be devoted to showing how we can construct such an order within the framework here. But for now, the vagueness of absolute adjectives must be stated as the conditional proposition in theorem 27.

**Theorem 27. Vagueness of absolute adjectives.** *For  $Q \in AA$ , if there is some way of constructing a non-trivial  $>_Q^x$  for  $x \in \{s, c, t\}$ , then  $Q$  is potentially vague.*

*Proof.* Consider the CC t-model  $M$  such that  $D = \{a, b, c, d\}$ . Consider  $X \in CC$  such that  $X = \{a, b, c, d\}$ . Suppose  $\llbracket Q \rrbracket_X^c = \{a, b\}$ . Suppose  $\sim_Q^x = \{\langle a, b \rangle, \langle b, c \rangle, \langle c, d \rangle\}$  + reflexivity and symmetry. Suppose furthermore that for some  $x \in \{s, c, t\}$ ,  $a >_Q^x b >_Q^x c >_Q^x d$  in  $M$ . Therefore,

1. **Clear Case:**  $a \in \llbracket Q \rrbracket_X^s$ .
2. **Clear Non-Case:**  $d \in \llbracket \text{not } Q \rrbracket_X^s$ .
3. **Sorites Series:** The sequence  $\langle a, b, c, d \rangle$ .

Therefore,  $Q$  is potentially vague. □

We saw that adjectives of both classes can be vague (provided we have some way of associating AAs with non-trivial scales), but how does this analysis fulfil the desiderata that was proposed in at the beginning of section 5?

1. **Unified Analysis:** Borderline cases etc. arise through the same mechanisms with both relative and absolute adjectives.
2. **Class Distinctions:** At some level, there must be a difference between RAs and AAs such that it is possible, in principle, to account for the non-vagueness-related data in section 2.
3. **Possible Precision:** It should be possible for absolute adjectives to be assigned a precise meaning in some context.

My analysis clearly satisfies 1) **Unified Analysis:** since the properties of vague language are created by the tolerant definitions, and these definitions are identical with both relative and absolute adjectives, the vagueness associated with *tall* and the vagueness associated with *bald* comes out as a single pragmatic phenomenon. However, at the same time, the analysis accounts for 2) **Class Distinctions.** There is a very clear distinction between relative and absolute predicates (absolute predicates are subject to the AAA), but note that this difference lies at the level of their semantics not pragmatics. Finally, the analysis can account for 3) **Possible Precision** in the following way: clearly the simple fact that tolerant and strict denotations are calculated on the basis of classical semantic denotations and contextually given indifference relations does not automatically mean that for every predicate  $P$  in

every situation,  $P$  will be vague. By assumption, indifference relations are reflexive (you're always indifferent from yourself), so all the members of a predicate's classical extension will always be in its tolerant extension. Although, in a given context, it is possible that  $\llbracket P \rrbracket^c \subset \llbracket P \rrbracket^t$ , it is not necessary. In contexts in which it is important to be very precise, that is, in contexts in which differences between individuals that are close to each other with respect to  $>_p^c$  have an import for our purposes,  $\sim_p^X$  might not relate individuals that are on either side of the borderline of  $\llbracket P/\text{not } P \rrbracket^c$ . In the models and comparison classes in which  $\sim_p^X$  respects the border of  $\llbracket P \rrbracket^c$ , the predicate will appear to 'sharpen up': it will lose its borderline cases and its boundaries will stop appearing fuzzy. Thus, in a situation in which people with a single hair are perceived as relevantly different from those with no hair (like in the Yul Brynner example),  $\sim_{bald}^X$  will not relate completely bald individuals with individuals that have any hair, and  $\llbracket bald \rrbracket_X^s = \llbracket bald \rrbracket_X^c = \llbracket bald \rrbracket_X^t$ . In other words, *bald* will be precise<sup>18</sup>.

Despite these advantages, we also saw that assigning a non-context-sensitive classical (i.e. semantic) denotation to absolute adjectives is problematic for explaining their appearance in comparative constructions. I therefore examine the semantics and pragmatics of absolute comparatives in the next section.

## 6 Absolute Comparatives

In the Klein-ian perspective, scales are constructed through looking at axiom-governed, comparison class-based variation in predicate denotations. But if, as I proposed in the previous section, the semantic denotation of an absolute constituent does not change across CCs, then how can such a constituent be gradable?

My answer to this question is that, while the semantic meaning of an AA does not vary according to comparison class, something else does. In particular, what can vary across CCs is which individuals are considered to be indifferent from one another, i.e., the  $\sim_Q^X$ s. For example, if I compare Homer Simpson, who has exactly two hairs, with Yul Brynner, the two would not be considered indifferent with respect to baldness (Homer has hair!). However, if I add Marge Simpson (who has a very large hairdo) into the comparison class, then Yul and Homer start looking much more similar, when it comes to baldness. Thus, I propose, it should be possible to order individuals with respect to how close to being completely bald they are by looking at in which comparison classes they are considered indifferent to completely bald people<sup>19</sup>.

In the next subsection, I present a set of axioms that constrain indifference relations between individuals across comparison classes. Then, in the spirit of [van Benthem \(1982\)](#) and [van Rooij \(2011a\)](#), I will show that these axioms will allow us to construct non-trivial strict weak orders from the tolerant (i.e. pragmatic) meanings of absolute predicates.

<sup>18</sup>This analysis also allows for relative adjectives to be 'sharpened up'; indeed, I argue in [Burnett \(2011a\)](#) that we do find precise uses of relative adjectives in some restricted contexts, and therefore that this aspect of the proposal is empirically necessary.

<sup>19</sup>This proposal is similar in spirit to one made by [Récanati \(2010\)](#) within [Laserson \(1999\)](#)'s *Pragmatic Halos* framework. However, the present analysis is very different in its execution and its implications for the structure of the lexicon. See [Burnett \(2012\)](#) for discussion.

### 6.1 Tolerant Axiom Set

I first start by adopting a straightforward generalization of the Klein-ian semantics for the classical interpretation of the comparative.

**Definition 28. Tolerant comparative.** For  $x, y \in D$  and a relative adjective  $P$ ,  $x >_p^t y$  iff there is some  $X \in CC$  such that  $x \in \llbracket P \rrbracket_X^t$  and  $y \notin \llbracket P \rrbracket_X^t$ .

Now I propose the following axioms dealing with indifference relations.

(54) **No Skipping (NS):** For  $X \in CC$ , if  $x \sim_Q^X y$  and there is some  $z \in X$  such that  $x \geq_Q^t z \geq_Q^t y$ , then  $x \sim_Q^X z$ .

Where  $x \geq_Q^t y$  iff  $x >_Q^t y$  or  $x \approx_Q^t y$ . Note that we define the equivalence relation  $\approx_Q^t$  as below:

**Definition 29. Equivalent.** ( $\approx$ ) For  $Q \in SA$ ,  $a, b \in D$ , and  $x \in \{t, c, s\}$ ,  $a \approx_Q^x b$  iff  $a \not>_Q^x b$  and  $b \not>_Q^x a$ .

**No Skipping** says that, if person A is indistinguishable from person B, and there's a person C lying in between persons A and B on the relevant (tolerant) scale, then A and C are also indistinguishable. As we will see in the next section, NS performs a very similar function to van Benthem's *No Reversal*.

We now have two axioms that talk about how indifference relations can change across comparison classes. I call these the *granularity* axioms.

(55) **Granularity.**

**G1:** If for some  $X \in CC$ ,  $x \sim_Q^X y$  and  $x >_Q^t y$  and for  $X' : X \subset X'$ ,  $x \not>_Q^{X'} y$ , then there is some  $z \in X' - X$  such that either

$-z \sim_Q^{X'} x$  and  $z >_Q^t x$  and there is no  $z' \in X$  such that  $z' >_Q^t x$  and  $z >_Q^t z'$ . **or**

$-y \sim_Q^{X'} z$  and  $y >_Q^t z$  and there is no  $z' \in X$  such that  $y >_Q^t z'$  and  $z' >_Q^t z$ .

**G2:** For some  $X, X' \in CC$ , if  $X \subset X'$  and  $x \not>_Q^X y$  and  $x \sim_Q^{X'} y$ , then  $\exists z \in X' - X : x \not>_Q^{X'} z$ .

**G1** says that, if person A is (tolerantly) greater than person B, and they are indifferent in some comparison class, and then, in a larger comparison class, they are no longer indifferent, then this is because we have added a person C that is indifferent to either A or B. If person C is indifferent from person A, then C is greater than A, but smaller than any other greater element. If person C is indifferent from person B, then C is smaller than B, but greater than any other smaller individual. The intuition behind this axiom is that the reason that the relation between A and B is undone is because C is perceived as being closer to A or B than A was to B.

**G2** says that, if person A and person B are distinguishable in one CC, X, and then there's another CC, X', in which they are indistinguishable, then there is some person C in X'-X that is distinguishable from person A. This axiom is similar in spirit to van



Bentham's *Upward Difference* in that it ensures that, if there is a contrast/distinction in one comparison class, the existence of contrast is maintained in all the larger CCs.

The final axiom that we need is *Minimal Difference*:

(56) **Minimal Difference. (MD)** If  $x >_Q^c y$ , then  $x \not\sim_Q^{\{x,y\}} y$ .

**Minimal Difference** says that, if, at the finest level of granularity, you would make a classical distinction between two individuals, then they are not indistinguishable at that level of granularity. MD is similar in spirit to van Bentham's *Downward Difference* because it allows us to preserve contrasts down to the smallest comparison classes.

In summary, I propose four axioms that constrain the definition of the  $\sim_Q$  relation across comparison classes: **No Skipping**, **Granularity 1**, **Granularity 2**, and **Minimal Difference**. In the next section, I show how, using these constraints, we can prove a number of important results about the tolerant denotations of comparatives formed from absolute predicates.

## 6.2 Tolerant Semi-Orders

Firstly, Minimal Difference ensures that classical absolute denotations are subsets of tolerant denotations:

**Theorem 30.** *If  $Q \in AA$ , then  $>_Q^c \subseteq >_Q^t$ .*

*Proof.* Let  $x, y \in D$  such that  $x >_Q^c y$  to show that  $x >_Q^t y$ . Since  $x >_Q^c y$ , there is some  $X \in CC$  such that  $x \in \llbracket Q \rrbracket_X^c$  and  $y \notin \llbracket Q \rrbracket_X^c$ . Now consider  $\{x, y\} \in CC$ . By downward difference,  $x \in \llbracket Q \rrbracket_{\{x,y\}}^c$  and  $y \notin \llbracket Q \rrbracket_{\{x,y\}}^c$ . By the definition of  $\llbracket \rrbracket^t$ ,  $x \in \llbracket Q \rrbracket_{\{x,y\}}^t$ . Furthermore, by Minimal Difference,  $x \not\sim_Q^{\{x,y\}} y$ . So  $y \notin \llbracket Q \rrbracket_{\{x,y\}}^t$ . By the definition of  $>_Q^t$  (definition 28),  $x >_Q^t y$ .  $\square$

This is a welcome result, since it shows that the classical and tolerant denotations of absolute comparatives are in the same relationship as the classical and tolerant denotations of the positive forms of the adjectives, namely inclusion (cf. Cobreros et al. (2010)'s Corollary 1).

Secondly, with only No Skipping, we can prove that van Bentham's No Reversal holds at the tolerant level. It is in this sense that, as I mentioned, NS can be viewed as the tolerant correspondent of *No Reversal*.

**Theorem 31. No Tolerant Reversal (T-NR):** *For  $X \in CC$ , if  $x \in \llbracket Q \rrbracket_X^t$  and  $y \notin \llbracket Q \rrbracket_X^t$ , then there is no  $X' \in CC$  such that  $y \in \llbracket Q \rrbracket_{X'}^t$  and  $x \notin \llbracket Q \rrbracket_{X'}^t$ .*

*Proof.* Suppose  $x \in \llbracket Q \rrbracket_X^t$  and  $y \notin \llbracket Q \rrbracket_X^t$ . Suppose, for a contradiction that there is an  $X' \in CC$  such that  $y \in \llbracket Q \rrbracket_{X'}^t$  and  $x \notin \llbracket Q \rrbracket_{X'}^t$ . Therefore,  $x >_Q^t y$  and  $y >_Q^t x$ . Furthermore, by assumption and the definition of  $\llbracket Q \rrbracket_X^t$ , there is some  $d \sim_Q^X x$  such that  $d \in \llbracket Q \rrbracket_X^c$ , and  $d \not\sim_Q^X y$ . Thus  $d >_Q^t y$  and so  $d >_Q^t y >_Q^t x$ . Since  $d \sim_Q^X x$ , by No Skipping,  $d \sim_Q^X y$ .  $\perp$   $\square$

Finally, with the complete axiom set, we can show that, for all  $Q \in AA$ ,  $>_Q^t$  is both irreflexive and transitive, which are two of the three properties that define a strict weak order. Irreflexivity follows straightforwardly from the definition of  $>_Q^t$ .

**Lemma 32. Irreflexivity.** For all  $x \in D$ ,  $x \not>_Q^t x$ .

*Proof.* Since it is impossible, for any  $X \in CC$ , for an element to be both in  $\llbracket Q \rrbracket_X^t$  and not in  $\llbracket Q \rrbracket_X^t$ , by definition 23,  $>_Q^t$  is irreflexive.  $\square$

**Lemma 33. Transitivity.** For all  $x, y, z \in D$ , if  $x >_Q^t y$  and  $y >_Q^t z$ , then  $x >_Q^t z$ .

*Proof.* Suppose  $x >_Q^t y$  and  $y >_Q^t z$  to show that  $x >_Q^t z$ . Then there is some  $X \in CC$  such that  $x \in \llbracket Q \rrbracket_X^t$  and  $y \notin \llbracket Q \rrbracket_X^t$ . Thus, there is some  $d \in \llbracket Q \rrbracket_X^c$  such that  $d \sim_Q^X x$ . Now consider  $X \cup \{z\}$ .

**Case 1:**  $X \cup \{z\} = X$ . Since  $x \in \llbracket Q \rrbracket_X^t$  and  $y \notin \llbracket Q \rrbracket_X^t$ , by the assumption that  $y >_Q^t z$  and theorem 31,  $z \notin \llbracket Q \rrbracket_X^t$ . So  $x >_Q^t z$ .  $\checkmark$

**Case 2:**  $X \subset X \cup \{z\}$ . Suppose, for a contradiction that  $x \notin \llbracket Q \rrbracket_{X \cup \{z\}}^t$ . Since  $d \in \llbracket Q \rrbracket_X^c$ , by the AAA,  $d \in \llbracket Q \rrbracket_{X \cup \{z\}}^c$ . So  $d \not\sim_Q^{X \cup \{z\}} x$ . Then, by G1, there is some  $a \in X \cup \{z\} - X$  such that either  $a \sim_Q^{X \cup \{z\}} d$  and  $a >_Q^t d$  and there is no  $z'$  such that  $z' >_Q^t d$  and  $a >_Q^t z'$ , or  $a \sim_Q^{X \cup \{z\}} x$  and  $x >_Q^t a$  and there is no  $z' : x >_Q^t z'$  and  $z' >_Q^t a$ . Since  $d \in \llbracket Q \rrbracket_D^c$ , there is no  $b : b >_Q^t d$ . So  $x >_Q^t a$  and  $x \sim_Q^{X \cup \{z\}} a$ . Since  $X \cup \{z\} - X = \{z\}$ ,  $x >_Q^t z$  and  $x \sim_Q^{X \cup \{z\}} z$ , and there is no  $z' : x >_Q^t z'$  and  $z' >_Q^t z$ . But,  $x >_Q^t y >_Q^t z$ .  $\perp$  So  $x \in \llbracket Q \rrbracket_{X \cup \{z\}}^t$ . Now suppose for a contradiction that  $z \in \llbracket Q \rrbracket_{X \cup \{z\}}^t$ . Then there is some  $d' \in \llbracket Q \rrbracket_{X \cup \{z\}}^c$  such that  $d' \sim_Q^{X \cup \{z\}} z$ . Since  $x >_Q^t y$ , by the AAA,  $y \notin \llbracket Q \rrbracket_{X \cup \{z\}}^c$ . So  $d' >_Q^c y$  and, by assumption and theorem 30,  $d' >_Q^t y >_Q^t z$ . Since  $d' \sim_Q^{X \cup \{z\}} z$ , by No Skipping,  $d' \sim_Q^{X \cup \{z\}} y$ . Since  $y \notin \llbracket Q \rrbracket_X^t$ ,  $d' \not\sim_Q^X y$ . So, by G2, there is some  $a \in X \cup \{z\}$  such that  $d' \not\sim_Q^{X \cup \{z\}} a$ . Since  $X \cup \{z\} - X = \{z\}$ ,  $d' \not\sim_Q^{X \cup \{z\}} z$ .  $\perp$  So  $z \notin \llbracket Q \rrbracket_{X \cup \{z\}}^t$ .  $\checkmark$   $\square$

However, although  $>_Q^t$  is irreflexive and transitive, it is not quite yet a strict weak order because it is not almost connected. For the AC counter-model, see Burnett (2012). Nevertheless, with these axioms,  $<_Q^t$  does instantiate a type of order that has been proposed to be relevant for the meaning of comparatives: a **semi-order**. The definition of *semi-order* given in Scott and Suppes (1958) is given in definition 34 (see also Luce (1956) for a more complicated definition).

**Definition 34. Semi-Order.** A semi-order is a structure  $\langle D, > \rangle$ , with  $>$  a binary relation on  $D$  that satisfies the following conditions:

1. **Irreflexivity**

For all  $x \in D$ ,  $x \not> x$ .

2. **Interval Order**

For all  $x, y, v, w$ , if  $x > y$  and  $v > w$ , then either  $x > w$  or  $v > y$ .

3. **Semi-Transitive**

For all  $x, y, z, v$ , if  $x > y$  and  $y > z$ , then either  $x > v$  or  $v > z$ .

As shown in lemma 32,  $>_Q^t$  is irreflexive. Furthermore, we can prove that  $>_Q^t$  satisfies the interval order property.

**Lemma 35. Interval Order.** *For all  $x, y, v, w \in D$ , if  $x >_Q^t y$  and  $v >_Q^t w$ , then either  $x >_Q^t w$  or  $v >_Q^t y$ .*

*Proof.* Suppose  $x >_Q^t y$  and  $v >_Q^t w$  and  $x \not>_Q^t w$  to show  $v >_Q^t y$ . Since  $v >_Q^t w$ , there is some  $X \in CC$  such that  $v \in \llbracket Q \rrbracket_X^t$  and  $w \notin \llbracket Q \rrbracket_X^t$ . Now consider  $X \cup \{x\}$ .

**Case 1:**  $X \cup \{x\} = X$ . Since  $x \not>_Q^t w$ , by theorem 31,  $x \notin \llbracket Q \rrbracket_{X \cup \{x\}}^t$ , so  $v >_Q^t x$ . Furthermore, since  $v >_Q^t x$  and  $x >_Q^t y$  and  $>_Q^t$  is transitive (lemma 33),  $v >_Q^t y$ .  $\checkmark$

**Case 2:**  $X \subset X \cup \{x\}$ . **Case 2a:**  $v \in \llbracket Q \rrbracket_{X \cup \{x\}}^t$ . Suppose for a contradiction that  $x \in \llbracket Q \rrbracket_{X \cup \{x\}}^t$ . Then there is some  $d' \in \llbracket Q \rrbracket_{X \cup \{x\}}^c$  such that  $d' \sim_Q^{X \cup \{x\}} x$ . Since  $v >_Q^t w$ , by the AAA,  $w, x \notin \llbracket Q \rrbracket_D^c$ . So  $d' >_Q^c w$ , and by theorem 30,  $d' >_Q^t w$ . Since  $x \not>_Q^t w$ ,  $d' >_Q^t w \geq^t x$ . So, by No Skipping,  $w \sim_Q^{X \cup \{x\}} d'$ . Since  $w \notin \llbracket Q \rrbracket_X^t$ ,  $d' \not\sim_Q^X w$ . So, by G2,  $d' \not\sim_Q^{X \cup \{x\}} x$ .  $\perp$  So  $x \notin \llbracket Q \rrbracket_{X \cup \{x\}}^t$ . Therefore,  $v >_Q^t x$ . Since  $x >_Q^t y$ , by the transitivity of  $>_Q^t$ ,  $v >_Q^t y$ .  $\checkmark$

**Case 2b:**  $v \notin \llbracket Q \rrbracket_{X \cup \{x\}}^t$ . Since  $v \in \llbracket Q \rrbracket_X^t$  there is some  $d \in \llbracket Q \rrbracket_X^c$  such that  $d \sim_Q^X v$ . By the AAA,  $d \in \llbracket Q \rrbracket_{X \cup \{x\}}^c$ . So  $d \not\sim_Q^{X \cup \{x\}} v$ . Therefore, by G1, there is some element in  $X \cup \{x\} - X$ , i.e. in  $\{x\}$ , i.e.  $x$ , such that either  $x >_Q^t d$  and  $x \sim_Q^t d$  and there is no  $z' \in X \cup \{x\}$  such that  $z' >_Q^t d$  and  $x > z'$ , or  $v >_Q^t x$  and  $v \sim_Q^{X \cup \{x\}} x$  and there is no  $z' : v >_Q^t z'$  and  $z' >_Q^t x$ . Since  $d \in \llbracket Q \rrbracket_D^c$ , there is no  $z' : z' >_Q^t d$ . Therefore,  $v >_Q^t x$ . Since  $v >_Q^t x$  and  $x >_Q^t y$ , then, by the transitivity of  $>_Q^t$ ,  $v >_Q^t y$ .  $\checkmark$

□

Finally, we can show that  $>_Q^t$  is semi-transitive.

**Lemma 36. Semi-Transitive.** *For all  $x, y, z, v \in D$ , if  $x >_Q^t y$  and  $y >_Q^t z$ , then either  $x >_Q^t v$  or  $v >_Q^t z$ .*

*Proof.* Suppose  $x >_Q^t y$  and  $y >_Q^t z$ , and suppose furthermore that  $v \not>_Q^t z$  to show  $x >_Q^t v$ . Since  $>_Q^t$  is transitive (lemma 33),  $x >_Q^t z$ . So there is some  $X \in CC$  such that  $x \in \llbracket Q \rrbracket_X^t$  and  $z \notin \llbracket Q \rrbracket_X^t$ . Now consider  $X \cup \{v\}$ .

**Case 1:**  $X \cup \{v\} = X$ . Since  $v \not>_Q^t z$ ,  $v \notin \llbracket Q \rrbracket_{X \cup \{v\}}^t$ .  $\checkmark$

**Case 2:**  $X \subset X \cup \{v\}$ . Suppose for a contradiction that  $x \notin \llbracket Q \rrbracket_{X \cup \{v\}}^t$ . Since  $x \in \llbracket Q \rrbracket_X^t$ , there is some  $d' \sim_Q^X x$  such that  $d' \in \llbracket Q \rrbracket_X^c$ . Since  $x \notin \llbracket Q \rrbracket_{X \cup \{v\}}^t$ ,  $d' \not\sim_Q^{X \cup \{v\}} x$ . Therefore, by G1, either  $v >_Q^t d'$  and  $v \sim_Q^{X \cup \{v\}} d'$  and there is no  $z' : v >_Q^t z' >_Q^t d'$ , or  $x >_Q^t v$  and  $x \sim_Q^{X \cup \{v\}} v$  and there is no  $z' : x >_Q^t z' >_Q^t v$ . Since  $d' \in \llbracket Q \rrbracket_D^c$ , there is no  $z' : z >_Q^t d'$ , so  $x >_Q^t v$  and  $x \sim_Q^{X \cup \{v\}} v$  and there is no  $z' : x >_Q^t z' >_Q^t v$ . However, by assumption,  $x >_Q^t y >_Q^t z$ , and since  $z \geq^t v$ ,  $x >_Q^t y >_Q^t v$ .  $\perp$  So  $x \in \llbracket Q \rrbracket_{X \cup \{v\}}^t$ .

Now suppose for a contradiction that  $v \in \llbracket Q \rrbracket_{X \cup \{v\}}^t$ . Then there is some  $d \in \llbracket Q \rrbracket_{X \cup \{v\}}^c$  such that  $d \sim_Q^{X \cup \{v\}} v$ . Since  $v \not>_Q^t z$  and  $y >_Q^t z$ , by the AAA,  $v, z \notin \llbracket Q \rrbracket_{X \cup \{v\}}^c$ . So  $d >_Q^c v$  and  $d >_Q^c z$ , and by theorem 30,  $d >_Q^t v$  and  $d >_Q^t z$ . Furthermore, since  $v \not>_Q^t z$ ,  $d >_Q^t z \geq^t v$ . So, by No Skipping, there is some  $z' \in X \cup \{v\} - X$  such

that  $d' \not\sim_Q^{X \cup \{v\}} z'$ . Since  $X \cup \{v\} - X = \{v\}$ ,  $d' \not\sim_Q^{X \cup \{v\}} v$ .  $\perp$  So  $v \notin \llbracket Q \rrbracket_{X \cup \{v\}}^t$  and  $x >_Q^t v$ .  $\checkmark$

□

We can now prove that  $>_Q^t$  is a semi-order.

**Theorem 37. Semi-Order.** *With the axioms No Skipping, Granularity 1, Granularity 2, and Minimal Difference,*

(57) For all  $Q \in AA$ ,  $>_Q^t$  is a semi-order.

*Proof.* Immediately from lemmas 32, 35, and 36. □

Semi-orders are slightly weaker relations than strict weak orders, and they have been argued (by van Rooij (2011a)) to be the type of scales lexicalized by the implicit comparative construction (58). Indeed, the axiom set just proposed may be an appropriate analysis of the implicit absolute comparative (59).

(58) John is tall compared to Peter.

(59) John is bald compared to Peter.

However, when it comes to the explicit comparative, particularly with respect to explaining the appearance of Sorites arguments with predicates like *bald*, we want something stronger. If *bald* really means “has zero hairs”, and the way that we compare individuals in terms of baldness is by counting and seeing who has the smaller number of hairs<sup>20</sup>, then we would like the *balder* relation be almost connected: if Peter is balder (has fewer hairs) than Marc and we consider Phil, either Phil has the same number of hairs or fewer than Marc, or he has the same number of hairs (or more) than Peter. Therefore, in the next section, I show how we can strengthen the constraints on the behaviour of indifference relations to get the desired result.

### 6.3 Tolerant Strict Weak Orders

In order for  $<_Q^t$  to have the ‘almost connectedness’ property, I propose to substitute the first granularity axiom (G1 in (55)) for a new version: G1’:

(60) **Granularity 1’ (G1’).**

If  $x \sim_P^X y$ , then for all  $X' : X \subseteq X'$ ,  $x \sim_{P'}^{X'} y$ .

**G1’** says that if person A and person B are indistinguishable in comparison class  $X$ , then they are indistinguishable in all supersets of  $X$ . This is meant to reflect the fact that the larger the domain is (i.e. the larger the comparison class is), the more things

<sup>20</sup>At this point, I leave open the possibility that, for at least some of these absolute comparatives, we might be interested in a weaker relation. For example, it seems relatively clear that a characterization of the meaning of the terms *bald* and *balder* the way they are used in everyday speech does not only involve counting number of hairs, but it also involves taking into account how the hair is arranged, what kind of coverage the top of the head gets, the thickness of the hair etc. Thus, it is conceivable, taking these criteria into consideration, that *balder* may not be almost-connected: Peter could be balder than Marc, but Phil has a hair-style such that he is incomparable to either Peter, Marc or both.

can cluster together; however, it does not allow indifference relations to be broken by the introduction of new individuals, in the way that G1 does.

However, using the complete axiom set  $\{NS, G1', G2, MD\}$ , we can show that, for all  $Q \in AA$ ,  $>_Q^t$  is a strict weak order. Irreflexivity follows in the same way as with G1, and we can also prove transitivity.

**Lemma 38. Transitivity.** *For all  $x, y, z \in D$ , if  $x >_Q^t y$  and  $y >_Q^t z$ , then  $x >_Q^t z$ .*

*Proof.* Suppose  $x >_Q^t y$  and  $y >_Q^t z$  to show that  $x >_Q^t z$ . Then there is some  $X \in CC$  such that  $x \in \llbracket Q \rrbracket_X^t$  and  $y \notin \llbracket Q \rrbracket_X^t$ . Thus, there is some  $d \in \llbracket Q \rrbracket_X^c$  such that  $d \sim_Q^X x$ . Now consider  $X \cup \{z\}$ . By the AAA and the assumption that  $x >_Q^t y$  and  $y >_Q^t z$ ,  $y, z \notin \llbracket Q \rrbracket_{X \cup \{z\}}^c$ .

**Case 1:**  $X \cup \{z\} = X$ . Since  $x \in \llbracket Q \rrbracket_X^t$  and  $z \notin \llbracket Q \rrbracket_X^t$ ,  $x >_Q^t z$ .  $\checkmark$

**Case 2:**  $X \subset X \cup \{z\}$ . Since  $X \subset X \cup \{z\}$  and  $d \sim_Q^X x$ , by G1',  $d \sim_Q^{X \cup \{z\}} x$ . By the AAA,  $d \in \llbracket Q \rrbracket_{X \cup \{z\}}^c$ . So  $x \in \llbracket Q \rrbracket_{X \cup \{z\}}^t$ . Suppose, for a contradiction that  $z \in \llbracket Q \rrbracket_{X \cup \{z\}}^t$ . Then there is some  $d' \in \llbracket Q \rrbracket_{X \cup \{z\}}^c$  such that  $d' \sim_Q^{X \cup \{z\}} z$ . By assumption and since  $y \notin \llbracket Q \rrbracket_X^c$ , by MD,  $d' >_Q^t y >_Q^t z$ . So by No Skipping,  $d' \sim_Q^{X \cup \{z\}} y$ . Since  $y \notin \llbracket Q \rrbracket_X^t$ ,  $d' \not\sim_Q^X y$ . So by G2, since  $X \cup \{z\} - X = \{z\}$ ,  $d' \not\sim_Q^{X \cup \{z\}} z$ .  $\perp$ . So  $z \notin \llbracket Q \rrbracket_{X \cup \{z\}}^t$ , and  $x >_Q^t z$ .  $\checkmark$

□

Finally, we can prove almost connectedness.

**Lemma 39. Almost Connected.** *For all  $x, y \in D$ , if  $x >_Q^t y$  then for all  $z \in D$ , either  $x >_Q^t z$  or  $z >_Q^t y$ .*

*Proof.* Let  $x >_Q^t y$  and  $z \not>_Q^t y$  to show  $x >_Q^t z$ .

**Case 1:**  $x \in \llbracket Q \rrbracket_D^c$ . Since  $x >_Q^t y$  and  $z \not>_Q^t y$ ,  $z \notin \llbracket Q \rrbracket_D^c$ . So  $x >_Q^c z$ , and, by theorem 30,  $x >_Q^t z$ .  $\checkmark$

**Case 2:**  $x \notin \llbracket Q \rrbracket_D^c$ . Since  $x >_Q^t y$ , there is some  $X \in CC$  such that  $x \in \llbracket Q \rrbracket_X^t$  and  $y \notin \llbracket Q \rrbracket_X^t$ . So there is some  $d \in \llbracket Q \rrbracket_X^c$  such that  $d \sim_Q^X x$ . Consider  $X \cup \{z\}$ . Since  $z \not>_Q^t y$ ,  $x, y, z \notin \llbracket Q \rrbracket_{X \cup \{z\}}^c$ . Since  $d \sim_Q^X x$ , by G1',  $d \sim_Q^{X \cup \{z\}} x$  and by the AAA,  $d \in \llbracket Q \rrbracket_{X \cup \{z\}}^c$ . So  $x \in \llbracket Q \rrbracket_{X \cup \{z\}}^t$ . Now suppose for a contradiction that  $z \in \llbracket Q \rrbracket_{X \cup \{z\}}^t$ . Then there is some  $d' \in \llbracket Q \rrbracket_{X \cup \{z\}}^c$  such that  $d' \sim_Q^{X \cup \{z\}} z$ . Since  $d' \in \llbracket Q \rrbracket_{X \cup \{z\}}^c$  and  $y \notin \llbracket Q \rrbracket_{X \cup \{z\}}^c$ ,  $d' >_Q^c y$ ; so by theorem 30,  $d' >_Q^t y$ . Furthermore, since, by assumption,  $z \not>_Q^t y$ ,  $y \geq_Q^t z$ . Since  $d' \geq_Q^t y \geq_Q^t z$  and  $d' \sim_Q^{X \cup \{z\}} z$ , by No Skipping,  $d' \sim_Q^{X \cup \{z\}} y$ . However, since  $y \notin \llbracket Q \rrbracket_X^t$ , and by the AAA,  $d' \in \llbracket Q \rrbracket_X^c$ ,  $d' \not\sim_Q^X y$ . Since  $X \subset X \cup \{z\}$  and  $d' \sim_Q^{X \cup \{z\}} y$ , by G2, there is some  $a \in X \cup \{z\} - X$  such that  $d' \not\sim_Q^{X \cup \{z\}} a$ . Since  $X \cup \{z\} - X = \{z\}$ ,  $d' \not\sim_Q^{X \cup \{z\}} z$ .  $\perp$  So  $z \notin \llbracket Q \rrbracket_{X \cup \{z\}}^t$  and  $x >_Q^t z$ .  $\checkmark$

□

We can now prove the main theorem of this section:

**Theorem 40.** *If  $Q \in AA$ ,  $<_Q^t$  is a strict weak order.*

*Proof.* Immediate from lemmas 32, 38 and 39.  $\square$

In summary, the axioms presented in this section allow us to extract a strict weak order from the behaviour of tolerant denotations across comparison classes. Using this order (the order associated with the tolerant meaning of absolute adjectives) we can finally prove a non-conditional version of theorem 27: If  $Q$  is an absolute adjective,  $Q$  is potentially vague.

**Theorem 41. Vagueness of absolute adjectives.** *If  $Q \in AA$ , then  $Q$  is potentially vague.*

*Proof.* Consider the CC t-model  $M$  such that  $D = \{a, b, c, d, e\}$ . Consider  $X \in CC$  such that  $X = \{a, b, c, d, e\}$ . Suppose  $\llbracket Q \rrbracket_X^c = \{a\}$ . Suppose  $\sim_Q^X = \{\langle a, b \rangle, \langle b, c \rangle, \langle c, d \rangle\} +$  reflexivity and symmetry for all pairs except  $\langle a, b \rangle$ <sup>21</sup>. Suppose furthermore that  $a >_Q^t b >_Q^t c >_Q^t d >_Q^t e$  in  $M$ . Therefore,

1. **Clear Case:**  $a \in \llbracket Q \rrbracket_X^s$ .
2. **Clear Non-Case:**  $d \in \llbracket \text{not } Q \rrbracket_X^s$ .
3. **Sorites Series:** The sequence  $\langle a, b, c, d \rangle$ .

Therefore,  $Q$  is potentially vague. □

Note that the element  $e$ , that is both at the bottom of the tolerant scale associated with  $Q$  and indifferent from both  $c$  and  $d$  is required to satisfy the axiom G2.

#### 6.4 Partial Adjectives

In this subsection, I will briefly examine extending my approach to the class of *partial absolute adjectives* like *dirty* and *wet*. I will not develop a full analysis here since, in section 2, I suggested that these adjectives do not form a lexical class distinct from relative adjectives. The tolerant meanings of relative comparatives will be examined in section 7. Nevertheless, it is perhaps of interest to explore what an analysis of the gradability of partial absolute adjectives might look like within Klein-ian TCS, should it later turn out that they need to be distinguished from relative adjectives in order to account for some natural language data.

With total absolute adjectives in the previous subsection, I derived scales through looking at how classically  $\neg Q$  individuals could be related to classically  $Q$  individuals by indifference relations in different comparison classes. Since the conditions for being part of the classical extension of a total AA are very strict, there was a lot of room to order classically  $\neg Q$  individuals with respect to how close they ‘approximate  $Q$ -ness’, (to quote Récanati (2010)). However, this exact strategy will not work with partial adjectives because the conditions for classically satisfying  $Q$  with these constituents are much more permissive. For example, it is generally said in the literature that, to be *dirty*, it is sufficient to be spotted with only a single speck of dirt. Thus, as shown in (61), comparatives can relate two individuals that are both classically dirty.

<sup>21</sup>The asymmetry here is necessary to ensure that  $a$  falls in the strict extension of  $Q$ . See Burnett (2012) for a discussion about this point and its consequences for the definition of the *total/partial* distinction in non-degree semantics.

(61) This shirt is dirtier than that shirt. But they're both dirty.

Since the classical truth conditions of partial adjectives are existential, there is only one kind of individual that will fail to classically satisfy *dirty*: an individual with zero specks of dirt. Thus, only the individuals occupying the bottom endpoint of the scale associated with *dirty* will not be part of *dirty*'s classical extension in every CC. Furthermore, as soon as we adopt an axiom like *Minimal Difference*, it is easy to see that the tolerant and classical scales will be equivalent ( $>_Q^t = >_Q^c$ ). So how can it be possible to form meaningful comparatives with partial absolute adjectives?

One answer to this question is to take into account the well-known observation (made by [Kamp and Rossdeutscher \(1994\)](#), among others) that total and partial adjectives are, in some sense, duals of each other: total adjectives are universal (you must have all degrees of cleanliness to be *clean*); whereas, partial adjectives are existential (you only need one degree of dirtiness to be *dirty*). Recall that, with total adjectives, it was the tolerant denotation of the comparative that was non-trivial, and that tolerant denotations are defined by means of existential quantification over indifference relations (62-a). However, we have also defined, following [Cobrerros et al. \(2010\)](#), the dual notion of a strict denotation (62-b) which involves universal quantification over indifference relations.

(62) **Tolerant/Strict Duality**

- a.  $\llbracket Q \rrbracket^t = \{x : \exists d : d \sim_Q x \text{ and } d \in \llbracket Q \rrbracket^c\}$
- b.  $\llbracket Q \rrbracket^s = \{x : \forall d : d \sim_Q x, d \in \llbracket Q \rrbracket^c\}$

Thus, in this system, it is possible for an individual to be classically  $Q$  but be indifferent to something that is classically  $\neg Q$ , i.e. it is possible for an individual to be in the classical extension of  $Q$  but not in its strict extension. What we might propose, then, is that, while some individuals will never be indifferent from classically  $\neg Q$  individuals, others might be so in some comparison classes. Thus, defining the strict denotation of a comparative as in definition 42, we might construct orders in a parallel manner to total absolute adjectives.

**Definition 42. Strict Comparative** ( $>_Q^s$ ). For  $Q \in SA$ ,  $x >_Q^s y$  iff there is some  $X \in CC$  such that  $x \in \llbracket Q \rrbracket_X^s$  and  $y \notin \llbracket Q \rrbracket_X^s$ .

However, a full analysis of the scales associated with partial adjectives is out of the scope of this work.

## 6.5 Summary

In summary, I presented a solution to the puzzle of the existence of absolute comparatives. I proposed that absolute adjectives have non-context-sensitive, and therefore non-gradable, semantics, but context-sensitive pragmatics. I proposed that the non-trivial scales associated with absolute constituents are actually their tolerant scales, and I showed how to construct these orders from a series of axioms governing the establishment of indifference relations across comparison classes in the spirit

of van Benthem (1982) and van Rooij (2011a). In the next section, I consider the tolerant denotations of relative comparatives.

## 7 Relative Comparatives

In this section, I define the tolerant meanings for relative comparatives like those in (63).

- (63) a. John is taller than Mary.  
b. This book is more expensive than that magazine.

Firstly, given that the morphology in both of these constructions is identical, I adopt the same definition of the tolerant denotation of a relative comparative as I proposed for an absolute comparative (64).

- (64) **Tolerant Denotation of Relative Comparatives:**  
For  $P \in RA$  and  $x, y \in D$ ,  $x >_p^t y$  iff there is some  $X \in \llbracket P \rrbracket_X^t$  and  $y \notin \llbracket P \rrbracket_X^t$ .

Recall furthermore that, as proposed in section 5, the classical semantics of relative adjectives is subject to the axiom set {No reversal, Upward difference, Downward difference}, i.e. they are associated with non-trivial scales at the classical semantic level ( $>_p^c$ ). In the next subsection, I examine what happens if we adopt the same axioms governing the tolerant meaning of RA comparatives as we did for AA comparatives.

### 7.1 NS, G1'/2, MD with Relative Adjectives

The scales constructed out of the classical semantic denotation of RAs form non-trivial strict weak orders; therefore, we might wonder what consequences having this more complex structure at the classical level might have for the scales constructed out of tolerant denotations. One might think that we would be able to see some reflexes of the structure created by van Benthem's axioms for  $>_p^c$  in the  $>_p^t$  relations. However, surprisingly<sup>22</sup>, despite the fact that they hold at the classical level, none of van Benthem's axioms are theorems at the tolerant level (see Burnett (2012) for proofs). To ensure that  $>_p^t$  is a coherent, comparative-like relation, we need to add some constraints. Recall the axioms that I proposed to characterize how indifference relations associated with absolute predicates are constrained across comparison classes.

<sup>22</sup>In fact, this is a difference between Klein-ian TCS and simple TCS, since classical validities imply tolerant validities ( $\models^c \Rightarrow \models^t$ ) in the latter system (cf. Cobreros et al. (2010)'s Corollary 1).



(65) **Proposed Axioms**a. **No Skipping (NS):**

For  $X \in CC$ , if  $x \sim_p^X y$  and there is some  $z \in X$  such that  $x \geq_p^t z \geq_p^t y$ , then  $x \sim_p^X z$ .

b. **Granularity 1' (G1'):**

If  $x \sim_p^X y$ , then for all  $X' : X \subseteq X'$ ,  $x \sim_{X'}^{X'}$   $y$ .

c. **Granularity 2 (G2):**

For  $X \subset X'$ : If  $x \not\sim_p^X y$  and  $x \sim_{X'}^{X'}$   $y$ , then  $\exists z \in X' - X : x \not\sim_{X'}^{X'}$   $z$ .

d. **Minimal Difference (MD):**

If  $x >_p^c y$ , then  $x \not\sim_p^{\{x,y\}}$   $y$ .

Indeed, we can first note that, if we adopt No Skipping, it allows us to prove the tolerant version of van Benthem's No Reversal, just like it did with absolute adjectives.

**Theorem 43. No Tolerant Reversal (T-NR):** For  $P \in RA$  and  $X \in CC$ , if  $x \in \llbracket P \rrbracket_X^t$  and  $y \notin \llbracket P \rrbracket_X^t$ , then there is no  $X' \in CC$  such that  $y \in \llbracket P \rrbracket_{X'}^t$  and  $x \notin \llbracket P \rrbracket_{X'}^t$ .

*Proof.* Suppose  $x \in \llbracket P \rrbracket_X^t$  and  $y \notin \llbracket P \rrbracket_X^t$ . Suppose, for a contradiction that there is an  $X' \in CC$  such that  $y \in \llbracket P \rrbracket_{X'}^t$  and  $x \notin \llbracket P \rrbracket_{X'}^t$ . Therefore,  $x >_p^t y$  and  $y >_p^t x$ . Furthermore, by assumption and the definition of  $\llbracket P \rrbracket_X^t$ , there is some  $d \sim_p^X x$  such that  $d \in \llbracket P \rrbracket_X^c$ , and  $d \not\sim_p^X y$ . Thus  $d >_p^t y$  and so  $d >_p^t y >_p^t x$ . Since  $d \sim_p^X x$ , by No Skipping,  $d \sim_p^X y$ .  $\perp$   $\square$

As we saw by theorem 40, the axioms in (65) generate non-trivial tolerant strict weak orders out of trivial classical orders with predicates of the AA class. So we might think that by adopting them with the RA class, we would arrive at tolerant relative scales with the same properties as their tolerant absolute counterparts. However, this would be naive. In fact, if we combine NR, UD, and DD with NS, G1/2, and MD, we arrive at  $>_p^t$  relations that are not even transitive. This fact is proven in Burnett (2012). In other words, adopting the same axiom set for relative and absolute adjectives allows for some strange and unintuitive relations to instantiate  $>_p^t$ . I therefore propose that relative adjectives are subject to a slightly different axiom set, which will be discussed below.

7.2 *A New Solution*

Instead, I propose that indifference relations associated with relative adjectives are constrained by **No Skipping**, **Minimal Difference**, **G1'/G2** and the new axiom: **Classical Equivalence**.

(66) **Classical Equivalence (CE).** If  $x \approx_p^c y$  and  $x \sim_p^X d$  and  $y \in X$ , for some  $X \in CC$  and some  $d \in X$ , then  $y \sim_p^X d$ .

**Classical Equivalence** says that individuals at the same level on the classical scale are indifferent from the same individuals. This axiom is very powerful, and with NS and MD, we can prove that  $>_p^c$  and  $>_p^t$  are equivalent.

**Theorem 44.** For all  $P \in RA$ ,  $<_p^c = <_p^t$ .

*Proof.*  $\subseteq$  Immediately from (the RA version of) theorem 30 (based on Minimal Difference).  $\supseteq$  Let  $x >_p^t y$  to show  $x >_p^c y$ . Suppose, for a contradiction, that  $x \not>_p^c y$ . By No Tolerant Reversal and theorem 30,  $x \approx_p^c y$ . Since  $x >_p^t y$ , there is some  $X \in CC$  such that  $x \in \llbracket P \rrbracket_X^t$  and  $y \notin \llbracket P \rrbracket_X^t$ . Since  $x \in \llbracket P \rrbracket_X^t$ , there is some  $d \sim_p^X x$  such that  $d \in \llbracket P \rrbracket_X^c$ . Since  $x \approx_p^c y$ , by CE,  $y \sim_p^X d$ . So  $y \in \llbracket P \rrbracket_X^t$ .  $\perp$  Therefore,  $<_p^c = <_p^t$ .  $\square$

### 7.3 Summary of the Analysis of Comparatives

In summary, in this section, I have made the following proposals concerning the semantic and pragmatic analysis of explicit comparative constructions.

1. Both relative and absolute comparatives have the same classical, tolerant and strict definitions.
2. Relative adjectives and Absolute adjectives are characterized by different semantic and pragmatic axioms.
  - **Semantics:** RAs obey van Benthem’s axioms. AAs obey the *absolute adjective axiom*.
  - **Pragmatics:** RAs obey NS, G1’/2, MD, and CE. AAs obey NS, G1’/2, and MD.

The analysis is schematized in table 3.

Axiom type	Axiom	Absolute	Relative
Semantic	No Reversal	( $\checkmark$ )	$\checkmark$
	Upward Difference	( $\checkmark$ )	$\checkmark$
	Downward Difference	( $\checkmark$ )	$\checkmark$
	<b>Absolute Adj. Axiom</b>	$\checkmark$	$\times$
Pragmatic	No Skipping	$\checkmark$	$\checkmark$
	Minimal Difference	$\checkmark$	$\checkmark$
	Granularity 1’/2	$\checkmark$	$\checkmark$
	<b>Classical Equivalence</b>	$\times$	$\checkmark$

Table 3: Semantic and pragmatic axioms for scalar adjectives

These constraints allow us to prove the following results:

- (67) **AA Results**
- a. Absolute classical scales are trivial strict weak orders.
  - b. Absolute tolerant scales are non-trivial strict weak orders.
- (68) **RA Results**
- a. Relative classical scales are non-trivial strict weak orders.
  - b. Relative tolerant scales are equivalent to relative classical scales.

We therefore have a non-degree solution to the two puzzles that we set out to solve: 1) A unified account of vagueness with RAs and AAs, and 2) The gradability of absolute comparatives.

However, I argue that the axiom sets in table 3 get us more than just answers to the questions posed so far in the paper. In the next section, I will show how, with the proposals made so far, we can generate the scale structure distinctions found in languages like English and, in fact, arrive at a deeper understanding of why such distinctions exist in natural language.

## 8 Deriving Scale Structure

The entirety of this paper has been devoted to the study of scalar/gradable adjectives; that is, those that can easily appear in the comparative construction. However, there is a third lexical class in the adjectival domain that we have not looked at yet: non-scalar adjectives. In this section, I show how we can derive the non-gradability and precision of non-scalar adjectives from a straightforward extension of the analysis presented in the previous section. For an analysis of how other scale-structure distinctions (open/closed; discrete/dense etc.) can be captured within this framework, see Burnett (2012).

Non-scalar adjectives do not naturally appear in the comparative (although, often, some coercion is possible).

- (69)
- a. #This algebra is more atomic than that one.
  - b. #This map is more geographical than that one.
  - c. #This number is more prime than that one.
  - d. #John is more dead than Peter.

Additionally, they are generally not vague. We can see that the ability to appear in the comparative (in other words, the ability to be gradable) and the ability to be vague are intimately related by looking at cases of scalar-coercion with non-scalar adjectives. For example, the predicate **hexagonal** in (70-a) is precise: either a shape in a textbook has six sides, or it doesn't. Furthermore, this adjective is non-scalar: (70-b) is bizarre, and, to the extent that it means anything, it means that this shape is hexagonal and that shape is not.

- (70)
- a. This shape in a geometry textbook is **hexagonal**.
  - b. #This shape in a geometry textbook is **more hexagonal** than that shape.

However, there are other tolerant uses of *hexagonal*. For example, as discussed by Austin (1962) and others, France can be loosely described as hexagonal, even though, clearly, its coastline does not even come close to forming six perfect sides.

- (71) France is hexagonal.

With this use of *hexagonal*, we see the appearance of the properties of vague language<sup>23</sup>: borderline cases (France is hexagonal, but what about Canada?), fuzzy boundaries (how many grooves do you have to have in order to stop being hexagonal?), and we can straightforwardly construct a Sorites series by counting the number

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<sup>23</sup>I thank David Nicolas for bringing this example to my attention.

of grooves/sides of an object. With this use of *hexagonal*, the comparative suddenly becomes much better: instead of being non-sensical, (72) is true, and seems to mean something like ‘France approximates a hexagonal shape more than Canada’.

(72) France is **more hexagonal** than Canada.

In other words, what we have done in (71) and (72) is that we coerced the precise-non-scalar adjective *hexagonal* into a scalar absolute adjective, where the scale that it is associated with is constructed out of its tolerant meaning.

In summary, I have argued, in line with what is universally presupposed in the literature on vagueness, that gradability and vagueness go hand in hand: Non-scalar adjectives are precise. If they happen to have a non-precise use, then they become gradable. In the next section, I give an analysis of this empirical observation.

### 8.1 Deriving Non-Scalar Adjectives

Recall that, in my analysis of the properties of scalar adjectives in sections 6 and 7, I proposed that the semantics of absolute adjectives is constrained by one extra axiom than the semantics of relative adjectives (the AAA), and I also proposed that the pragmatics of relative adjectives is constrained by an extra axiom than the pragmatics of absolute adjectives (the CE axiom). Thus, in a certain sense, absolute predicates have a more restrictive semantic meaning, but a looser pragmatic meaning than do relative adjectives.

I propose that this analysis has the potential to yield a straightforward account of why non-scalar adjectives are neither gradable nor vague. In particular, I propose that members of the non-scalar (NS) class have the most restrictive meaning of all the types of adjectives: they are subject to strong constraints on both their semantics and their pragmatics. In other words, I propose that non-scalar adjectives obey both the AAA and the CE axiom. Thus, NS adjectives can be viewed as hybrids between relative and absolute adjectives, as shown in table 4.

Axiom type	Axiom	Absolute	Non-Scalar	Relative
Semantic	No Reversal	(✓)	(✓)	✓
	Upward Difference	(✓)	(✓)	✓
	Downward Difference	(✓)	(✓)	✓
	<b>Absolute Adj. Axiom</b>	✓	✓	×
Pragmatic	No Skipping	✓	✓	✓
	Minimal Difference	✓	✓	✓
	Granularity 1/2	✓	✓	✓
	<b>Classical Equivalence</b>	×	✓	✓

Table 4: Semantic and pragmatic axioms for (non-)scalar adjectives

This view of non-scalar adjectives yields clear predictions about whether or not they can be licensed in the comparative and whether or not they are potentially vague. Firstly, we can show that non-scalar adjectives have trivial classical scales, just like absolute adjectives.

**Theorem 45.** *If  $S \in NS$ , then  $>_S^c$  is homomorphic to the two-element boolean algebra.*

*Proof.* Immediately, because  $S$  obeys the AAA. (See theorem 20.) □

Secondly, we can show that classical scales associated with NS adjectives are identical to their tolerant scales, just like relative adjectives.

**Theorem 46.** *If  $S \in NS$ , then  $>_S^c = >_S^t$ .*

*Proof.*  $\subseteq$  Show if  $x >_S^c y$ , then  $x >_S^t y$ . Immediately from (a NS version of) theorem 30.  $\supseteq$  Suppose  $x >_S^t y$  to show  $x >_S^c y$ . Then there is some  $X \in CC$  such that  $x \in \llbracket S \rrbracket_X^t$  and  $y \notin \llbracket S \rrbracket_X^t$ . Since  $y \notin \llbracket S \rrbracket_X^t$ , but the AAA,  $y \notin \llbracket S \rrbracket_X^c$ . Suppose, for a contradiction, that  $x \notin \llbracket S \rrbracket_X^c$ . Therefore  $x \approx_S^c y$ . Also, there is some  $x \sim_S^X d$  such that  $d \in \llbracket S \rrbracket_S^c$ . Since  $x \approx_S^c y$  and  $x \sim_S^X d$ , by CE,  $y \sim_S^X d$ . So  $y \in \llbracket S \rrbracket_X^t$ .  $\perp$  So  $x \in \llbracket S \rrbracket_X^c$ , and  $x >_S^c y$ . □

A corollary of theorem 46 is that non-scalar adjectives also have trivial tolerant scales.

**Corollary 47.** *If  $S \in NS$ , then  $<_S^t$  is homomorphic to the two-element boolean algebra.*

*Proof.* Immediately from theorem 45 and theorem 46. □

Assuming that being licensed in the comparative requires a non-trivial scale at some level of meaning, we correctly predict that NS predicates should be infelicitous in this syntactic construction. Furthermore, another corollary of theorem 46 is that non-scalar adjectives are not potentially vague.

**Corollary 48.** *If  $S$  is a non-scalar adjective, then  $S$  is not potentially vague.*

*Proof.* Since, as shown by theorem 46, for all  $x \in \{t, c, s\}$ ,  $>_S^x$  is homomorphic to 2, there is not model  $M$  such that a Soritical series can be formed in  $M$ . Therefore  $S$  is not potentially vague. □

In summary, I presented a novel analysis of the lack of gradability and the lack of vagueness of non-scalar adjectives. I argue that this approach has an advantage over other approaches in the literature, particularly the approach to non-scalarity adopted by degree semanticists. In this framework, non-scalar adjectives are analyzed as lacking a degree argument; however, no explanation is ever given for why the argument structure of certain adjectival predicates may vary in this way. However, in my analysis, both the precision and the non-gradability of non-scalar adjectives are derived from constraints that are independently needed to account for relative and absolute adjectival predicates. Thus, we have a new vision of the members of the NS class, not as deficient in their selectional properties, but rather as the missing link between the absolute and relative lexical classes.

## 9 Conclusion

In conclusion, in this paper, I have argued for a number of proposals concerning vagueness, “imprecision”, and the semantics and pragmatics of (non)scalar adjectives. The principal ones are the following:

1. Both relative and absolute adjectives have precise semantics; however, the classical denotations of relative adjectives are context-sensitive and the classical denotations of absolute constituents are not context sensitive.
2. Vagueness is pragmatic: the puzzling properties of vague language arise due to pragmatic reasoning processes that apply to relative and absolute adjectives in the same way.
3. The non-trivial scale associated with absolute comparatives, i.e. what allows AAs to appear in the comparative construction, is constructed out of the positive form’s context-sensitive tolerant denotation. We can constrain how tolerant denotations can change across comparison classes such that the resulting comparative relation is a strict weak order.
4. Non-scalar adjectives are subject to the semantic constraints of absolute adjectives and the pragmatic constraints of relative adjectives. Therefore, they have trivial classical and tolerant scales. Due to this fact, they cannot appear in the comparative and are not potentially vague.

In addition, I have made some proposals about how to model the scale-structure distinctions that have traditionally been made only in degree semantics within a delineation framework. Thus, one way of viewing some of the contributions in this article is as a response to a challenge raised by [Kennedy \(2007\)](#) (p.41) for degree-free approaches. He says,

In particular, an analysis that derives gradability from a general, non-scalar semantics for vague predicates must explain the empirical phenomena that have been the focus of this paper: the semantic properties of relative and absolute gradable adjectives in the positive form. While it may be difficult but not impossible to explain some of these features, I do not see how such an approach can account for the basic facts of the relative/absolute distinction in a non-stipulative way. . . the challenge for a non-degree-based analysis is to explain why only relative adjectives are vague in the positive form, while absolute adjectives have fixed positive and negative extensions, but remain fully gradable.

In this paper, I have shown that the puzzles raised by absolute adjectives for a theory of vagueness and comparison can be solved within a Klein-ian framework, provided that we have an appropriate account of the puzzling features of vague language. Furthermore, I have shown that the scale-structure properties that have traditionally been the exclusive domain of degree semantics can arise naturally from certain intuitive statements about how individuals can and cannot be indifferent across comparison classes.

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